

Experiment

A process that produces a single outcome whose result cannot be predicted with certainty.

Sample Space

The collection of all outcomes that can result from a selection, decision, or experiment.

Events and Sample Space As discussed in Chapter 1, data come in many forms and are gathered in many ways. In a business environment, when a sample is selected or a decision is made, there are generally many possible outcomes. In probability language, the process that produces the outcomes is an **experiment**. In business situations, the experiment can range from an investment decision to a personnel decision to a choice of warehouse location.

For instance, a very simple experiment might involve flipping a coin one time. When this experiment is performed, two possible experimental outcomes can occur: head and tail. If the coin-tossing experiment is expanded to involve two flips of the coin, the experimental outcomes are

Head on first flip and Head on second flip, denoted by (H,H)

Head on first flip and tail on second flip, denoted by (H,T)

Tail on first flip and Head on second flip, denoted by (T,H)

Tail on first flip and Tail on second flip, denoted by (T,T)

The collection of possible experimental outcomes is called the **sample space**.

EXAMPLE 4-1 Defining the Sample Space

TRY PROBLEM 4.1

Able Accounting A partner for Able Accounting, a large regional accounting firm, is analyzing the performance of her many audit teams. She is particularly interested in whether the audits are finished by the projected completion date. She is interested in determining the sample space (possible outcomes) for two randomly selected audits. To do this, she can use the following steps:

Step 1 Define the experiment.

The experiment is the audit. Of interest is the status of an audit completion.

Step 2 Define the outcomes for one trial of the experiment.

The partner can define the outcomes to be

e_1 = Audit done early

e_2 = Audit done on time

e_3 = Audit done late

Step 3 Define the sample space.

The sample space (SS) for an experiment involving a single audit is

$$SS = \{e_1, e_2, e_3\}$$

If the experiment is expanded to include two audits, the sample space is

$$SS = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9\}$$

where the outcomes include what happens on both audits and are defined as

Outcome	Audit 1	Audit 2
e_1	early	early
e_2	early	on time
e_3	early	late
e_4	on time	early
e_5	on time	on time
e_6	on time	late
e_7	late	early
e_8	late	on time
e_9	late	late

TRY PROBLEM 4.1

EXAMPLE 4-2 Defining the Sample Space

Lincoln Marketing Research Recently, Lincoln Marketing Research in Lincoln, Nebraska, was retained to interview television viewers to determine whether they objected to having ads for hard liquor on TV. The analyst assigned to the project is interested in listing the sample space (possible outcomes). To do this, he can use the following steps:

Step 1 Define the experiment.

The experiment involves selecting three television viewers and posing the question: “Would you object to hard-liquor advertisements on television?”

Step 2 Determine the outcome for a single trial of the simple experiment.

The possible outcomes when one person is interviewed are

no
yes

Step 3 Define the sample space.

If three people are interviewed (3 trials), the sample space (possible outcomes) is

Outcome		Viewer 1	Viewer 2	Viewer 3
e_1	=	no	no	no
e_2	=	no	no	yes
e_3	=	no	yes	no
e_4	=	no	yes	yes
e_5	=	yes	no	no
e_6	=	yes	no	yes
e_7	=	yes	yes	no
e_8	=	yes	yes	yes

For instance, one possible experimental outcome is e_1 , all three people say no. Thus this outcome is (no, no, no).

Using Tree Diagrams A tree diagram is often a useful way to define the sample space for an experiment that helps ensure that no outcomes are omitted. Example 4-3 illustrates how a tree diagram is used.

TRY PROBLEM 4.2

EXAMPLE 4-3 Using a Tree Diagram to Define the Sample Space

Lincoln Marketing Research In Example 4-2, Lincoln Marketing Research was involved in a project in which television viewers were asked whether they objected to hard-liquor advertisements being shown on television. The analyst is interested in listing the sample space, using a tree diagram as an aid, when three viewers are interviewed. The following steps can be used:

Step 1 Define the experiment.

Three people are interviewed and asked, “Would you object to hard-liquor advertisements on television?” Thus, the experiment consists of three trials.

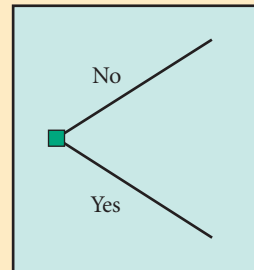
Step 2 Define the outcomes for a single trial of the experiment.

The possible outcomes when one person is interviewed are

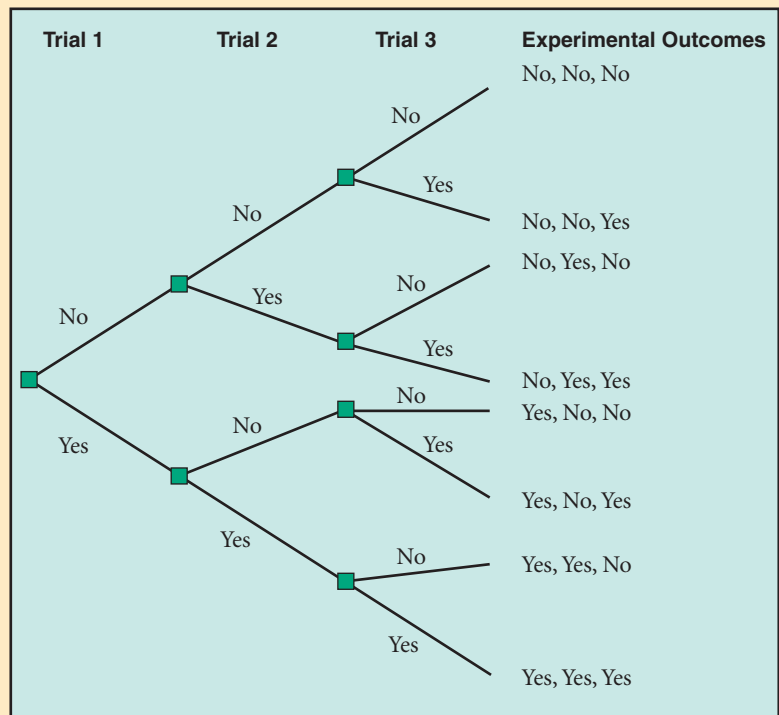
no
yes

Step 3 Define the sample space for three trials using a tree diagram.

Begin by determining the outcomes for a single trial. Illustrate these with tree branches beginning on the left side of the page:



For each of these branches, add branches depicting the outcomes for a second trial. Continue until the tree has the number of sets of branches corresponding to the number of trials.

**Event**

A collection of experimental outcomes.

A collection of possible outcomes is called an **event**. An example will help clarify these terms.

EXAMPLE 4-4 Defining an Event of Interest

Able Accounting The Able Accounting firm in Example 4-1 is interested in the sample space for an audit experiment in which the outcome of interest is the audit's completion status. The sample space is the list of all possible outcomes from the experiment. The accounting firm is also interested in specifying the outcomes that make up an event of interest. This can be done using the following steps:

Step 1 Define the experiment.

The experiment consists of two randomly chosen audits.

TRY PROBLEM 4.10

Step 2 List the outcomes associated with one trial of the experiment.

For a single audit the following completion-status possibilities exist:

- Audit done early
- Audit done on time
- Audit done late

Step 3 Define the sample space.

For two audits (two trials), we define the sample space:

Experimental Outcome		Audit 1	Audit 2
e_1	=	early	early
e_2	=	early	on time
e_3	=	early	late
e_4	=	on time	early
e_5	=	on time	on time
e_6	=	on time	late
e_7	=	late	early
e_8	=	late	on time
e_9	=	late	late

Step 4 Define the event of interest.

The event of interest at least one audit is completed late is composed of all the outcomes in which one or more audits are late. This event (E) is

$$E = \{e_3, e_6, e_7, e_8, e_9\}$$

There are five ways in which one or more audits are completed late.

Mutually Exclusive Events

Two events are mutually exclusive if the occurrence of one event precludes the occurrence of the other event.

Mutually Exclusive Events Keeping in mind the definitions for *experiment*, *sample space*, and *events*, we introduce two additional concepts. The first is **mutually exclusive events**.

ABLE ACCOUNTING (CONTINUED) Consider again the Able Accounting firm example. The possible outcomes for two audits are

Experimental Outcomes		Audit 1	Audit 2
e_1	=	early	early
e_2	=	early	on time
e_3	=	early	late
e_4	=	on time	early
e_5	=	on time	on time
e_6	=	on time	late
e_7	=	late	early
e_8	=	late	on time
e_9	=	late	late

Suppose we define one event as consisting of the outcomes in which at least one of the two audits is late.

$$E_1 = \{e_3, e_6, e_7, e_8, e_9\}$$

Further, suppose we define two more events as follows:

$$E_2 = \text{neither audit is late} = \{e_1, e_2, e_4, e_5\}$$

$$E_3 = \text{both audits are finished at the same time} = \{e_1, e_5, e_9\}$$

Events E_1 and E_2 are mutually exclusive: If E_1 occurs, E_2 cannot occur; if E_2 occurs, E_1 cannot occur. That is, if at least one audit is late, then it is not possible for neither audit to be late. We can verify this fact by observing that no outcomes in E_1 appear in E_2 . This observation provides another way of defining mutually exclusive events: Two events are mutually exclusive if they have no common outcomes.

Independent and Dependent Events A second probability concept is that of **independent** versus **dependent** events.

Business Application

Independent Events

Two events are independent if the occurrence of one event in no way influences the probability of the occurrence of the other event.

Dependent Events

Two events are dependent if the occurrence of one event impacts the probability of the other event occurring.

MOBILE EXPLORATION Mobile Exploration is a subsidiary of the Mobile Corporation and is responsible for oil and natural gas exploration worldwide. During the exploration phase, seismic surveys are conducted that provide information about the earth's underground formations. Based on past history, the company knows that if the seismic readings are favorable, oil or gas more likely will be discovered than if the seismic readings are not favorable. However, the readings are not perfect indicators. Suppose the company currently is exploring in the eastern part of Australia. The possible outcomes for the seismic survey are defined as

$$e_1 = \text{favorable}$$

$$e_2 = \text{unfavorable}$$

If the company decides to drill, the outcomes are defined as

$$e_3 = \text{strike oil or gas}$$

$$e_4 = \text{dry hole}$$

If we let the event A be that the seismic survey is favorable and event B be that the hole is dry, we can say that the events A and B are not mutually exclusive, because if one event occurs, it does not preclude the other event from occurring. We can also say that the two events are dependent because the chance of a dry hole depends on whether the seismic survey is favorable or unfavorable.

EXAMPLE 4-5 Mutually Exclusive Events

TRY PROBLEM 4.8

Barcelona Assembly Barcelona Assembly, located in Barcelona, Spain, does contract assembly work for Hewlett-Packard. Each item produced on the assembly line can be thought of as an experimental trial. The managers at this facility can analyze their process to determine whether the events of interest are mutually exclusive using the following steps:

Step 1 Define the experiment.

The experiment is producing a part on an assembly line.

Step 2 Define the outcomes for a single trial of the experiment.

On each trial the outcome is either a *good* or a *defective* item.

Step 3 Define the sample space.

If two products are produced (two trials), the following sample space is defined:

Experimental Outcomes	
Product 1	Product 2
$e_1 = \text{good}$	good
$e_2 = \text{good}$	defective
$e_3 = \text{defective}$	good
$e_4 = \text{defective}$	defective