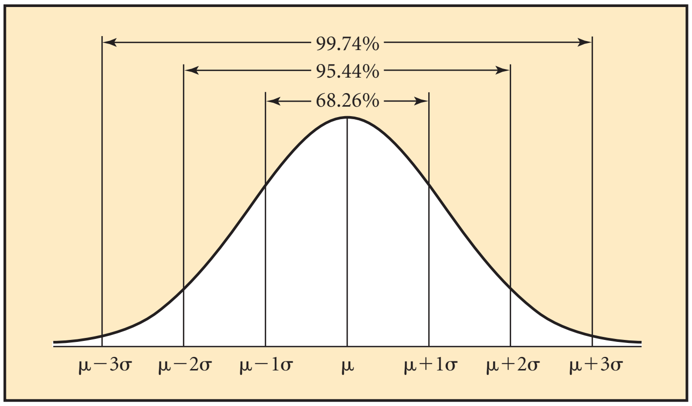
FIGURE 6.11

Approximate Areas Under the Normal Curve



6-1: Exercises

Skill Development

6-1. A population is normally distributed with $\mu = 100$ and $\sigma = 20$.

a. Find the probability that a value randomly selected from this population will have a value greater than 130.

b. Find the probability that a value randomly selected from this population will have a value less than 90.

c. Find the probability that a value randomly selected from this population will have a value between 90 and 130.

6-2. For a standardized normal distribution, calculate the following probabilities:

a. $P(0.00 < z \leq 2.33)$

b. $P(-1.00 < z \leq 1.00)$

c. $P(1.78 < z < 2.34)$

6-3. For a normally distributed population with $\mu = 200$ and $\sigma = 20$, determine the standardized z-value for each of the following:

a. $x = 225$

b. $x = 190$

c. $x = 240$

6-4. For a standardized normal distribution, calculate the following probabilities:

a. $P(z < 1.5)$

b. $P(z \geq 0.85)$

c. $P(-1.28 < z < 1.75)$

6-5. A random variable is known to be normally distributed with the following parameters:

$$\mu = 5.5 \quad \text{and} \quad \sigma = .50$$

a. Determine the value of x such that the probability of a value from this distribution exceeding x is at most 0.10.

b. Referring to your answer in part a, what must the population mean be changed to if the probability of exceeding the value of x found in part a is reduced from 0.10 to 0.05?

6-6. For a standardized normal distribution, determine a value, say z_0, so that

a. $P(0 < z < z_0) = 0.4772$

b. $P(-z_0 \leq z < 0) = 0.45$

c. $P(-z_0 \leq z \leq z_0) = 0.95$

d. $P(z > z_0) = 0.025$

e. $P(z \leq z_0) = 0.01$

6-7. Assume that a random variable is normally distributed with a mean of 1,500 and a variance of 324.

a. What is the probability that a randomly selected value will be greater than 1,550?

b. What is the probability that a randomly selected value will be less than 1,485?

c. What is the probability that a randomly selected value will be either less than 1,475 or greater than 1,535?

6-8. A randomly selected value from a normal distribution is found to be 2.1 standard deviations above its mean.

a. What is the probability that a randomly selected value from the distribution will be greater than 2.1 σ_s above the mean?

b. What is the probability that a randomly selected value from the distribution will be less than 2.1 σ from the mean?

6-9. A random variable is normally distributed with a mean of 60 and a standard deviation of 9.

a. What is the probability that a randomly selected value from the distribution will be less than 46.5?

b. What is the probability that a randomly selected value from the distribution will be greater than 78?

c. What is the probability that a randomly selected value will be between 51 and 73.5?

6-10. A random variable is normally distributed with a mean of 25 and a standard deviation of 5. If an observation is randomly selected from the distribution:

a. What value will be exceeded 10% of the time?
b. What value will be exceeded 85% of the time?
c. Determine two values of which the smallest has 25% of the values below it and the largest has 25% of the values above it.
d. What value will 15% of the observations be below?

6-11. Consider a random variable, z, that has a standardized normal distribution. Determine the following probabilities:

a. $P(0 < z < 1.96)$
b. $P(z > 1.645)$
c. $P(1.28 < z \leq 2.33)$
d. $P(-2 \leq z \leq 3)$
e. $P(z > -1)$

6-12. For the following normal distributions with parameters as specified, calculate the required probabilities:

a. $\mu = 5, \sigma = 2$; calculate $P(0 < x < 8)$.
b. $\mu = 5, \sigma = 4$; calculate $P(0 < x < 8)$.
c. $\mu = 3, \sigma = 2$; calculate $P(0 < x < 8)$.
d. $\mu = 4, \sigma = 3$; calculate $P(x > 1)$.
e. $\mu = 0, \sigma = 3$; calculate $P(x > 1)$.

6-13. A random variable, x, has a normal distribution with $\mu = 13.6$ and $\sigma = 2.90$. Determine a value, x_0, so that

a. $P(x > x_0) = 0.05$.
b. $P(x \leq x_0) = 0.975$.
c. $P(\mu - x_0 \leq x \leq \mu + x_0) = 0.95$.

Business Applications

6-14. The average number of acres burned by forest and range fires in a large New Mexico county is 4,300 acres per year, with a standard deviation of 750 acres. The distribution of the number of acres burned is normal.

a. Compute the probability in any year that more than 5,000 acres will be burned.
b. Determine the probability in any year that fewer then 4,000 acres will be burned.
c. What is the probability that between 2,500 and 4,200 acres will be burned?
d. In those years when more than 5,500 acres are burned, help is needed from eastern-region fire teams. Determine the probability help will be needed in any year.

6-15. In the National Weekly Edition of the *Washington Post* (December 19–25, 2005), firstStreet, Inc. advertised an atomic digital watch from LaCrosse Technology. It is radio-controlled and maintains its accuracy by reading a radio signal from a WWVB radio signal from Colorado. It neither loses nor gains a second in 20 million years. It is powered by a 3-volt lithium battery expected to last three years. Suppose the life of the battery has a standard deviation of 0.3 years and is normally distributed.

a. Determine the probability that the watch's battery will last longer than $3\frac{1}{2}$ years.
b. Calculate the probability that the watch's battery will last more than 2.75 years.
c. Compute the length-of-life value for which 10% of the watch's batteries last longer.

6-16. A global financial institution transfers a large data file every evening from offices around the world to its New York City headquarters. Once the file is received, it must be cleaned and partitioned before being stored in the company's data warehouse. Each file is the same size and the time required to transfer, clean, and partition a file is normally distributed, with a mean of 1.5 hours and a standard deviation of 15 minutes.

a. If one file is selected at random, what is the probability that it will take longer than 1 hour and 55 minutes to transfer, clean, and partition the file?
b. If a manager must be present until 85% of the files are transferred, cleaned, and partitioned, how long will the manager need to be there?
c. What percentage of the data files will take between 63 minutes and 110 minutes to be transferred, cleaned, and partitioned?

6-17. Bowser Bites Industries (BBI) sells large bags of dog food to warehouse clubs. BBI uses an automatic filling process to fill the bags. Weights of the filled bags are approximately normally distributed with a mean of 50 kilograms and a standard deviation of 1.25 kilograms.

a. What is the probability that a filled bag will weigh less than 49.5 kilograms?
b. What is the probability that a randomly sampled filled bag will weigh between 48.5 and 51 kilograms?
c. What is the minimum weight a bag of dog food could be and remain in the top 15% of all bags filled?
d. BBI is unable to adjust the mean of the filling process. However, it is able to adjust the standard deviation of the filling process. What would the standard deviation need to be so that no more than 2% of all filled bags weigh more than 52 kilograms?

6-18. T & S Industries manufactures a wash-down motor that is used in the food processing industry. The motor is marketed with a warranty that guarantees it will be replaced free of charge if it fails within the first 13,000 hours of operation. On the average, T & S wash-down motors operate for 15,000 hours with a standard deviation of 1,250 hours before failing. The number of operating hours before failure is approximately normally distributed.

a. What is the probability that a wash-down motor will have to be replaced free of charge?
b. What percentage of T & S wash-down motors can be expected to operate for more than 17,500 hours?
c. If T & S wants to design a wash-down motor so that no more than 1% are replaced free of charge, what would the average hours of operation before failure have to be if the standard deviation remains at 1,250 hours?

6-19. An Internet retailer stocks a popular electronic toy at a central warehouse that supplies the eastern United States. Every week the retailer makes a decision about how many units of the toy to stock. Suppose that weekly demand for the toy is approximately normally distributed with a mean of 2,500 units and a standard deviation of 300 units.

a. If the retailer wants to limit the probability of being out of stock of the electronic toy to no more than 2.5% in a week, how many units should the central warehouse stock?
b. If the retailer has 2,750 units on hand at the start of the week, what is the probability that weekly demand will be greater than inventory?
c. If the standard deviation of weekly demand for the toy increases from 300 units to 500 units, how many more toys would have to be stocked to ensure that the probability of weekly demand exceeding inventory is no more than 2.5%?

6-20. A private equity firm is evaluating two alternative investments. Although the returns are random, each investment's return can be described using a normal distribution. The first investment has a mean return of $2,000,000 with a standard deviation of $125,000. The second investment has a mean return of $2,275,000 with a standard deviation of $500,000.

a. How likely is it that the first investment will return $1,900,000 or less?
b. How likely is it that the second investment will return $1,900,000 or less?
c. If the firm would like to limit the probability of a return being less than $1,750,000, which investment should it make?

6-21. A-1 Plumbing and Repair provides customers with firm quotes for a plumbing repair job before actually starting the job. In order to be able to do this, A-1 has been very careful to maintain time records over the years. For example, it has determined that the time it takes to remove a broken sink disposal and install a new unit is normally distributed with a mean equal to 47 minutes and a standard deviation equal to 12 minutes. The company bills at $75.00 for the first 30 minutes and $2.00 per minute for anything beyond 30 minutes.

Suppose the going rate for this procedure by other plumbing shops in the area is $85.00, not including the cost of the new equipment. If A-1 bids the disposal job at $85, on what percentage of such jobs will the actual time required exceed the time for which it will be getting paid?

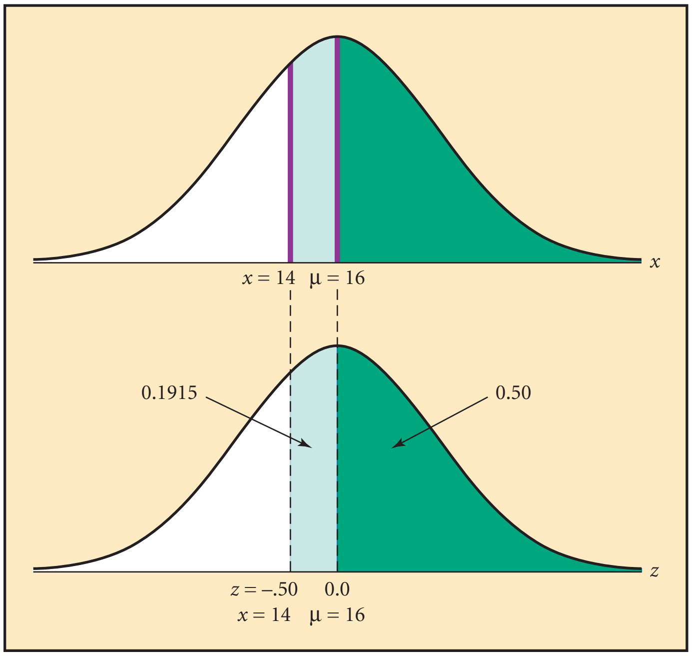
6-22. J.J. Kettering & Associates is a financial planning group in Fresno, California. The company specializes in doing financial planning for schoolteachers in the Fresno area. As such, it administers a 403(b) tax shelter annuity program in which public schoolteachers can participate. The teachers can contribute up to $20,000 per year on a pretax basis to the 403(b) account. Very few teachers have incomes sufficient to allow them to make the maximum contribution. The lead analyst at J.J. Kettering & Associates has recently analyzed the company's 403(b) clients and determined that the annual contribution is approximately normally distributed with a mean equal to $6,400. Further, he has determined that the probability a customer will contribute over $13,000 is 0.025. Based on this information, what is the standard deviation of contributions to the 403(b) program?

6-23. A senior loan officer for Wells Fargo Bank has recently studied the bank's real estate loan portfolio and found that the distribution of loan balances is approximately normally distributed with a mean of $155,600 and a standard deviation equal to $33,050. As part of an internal audit, bank auditors recently randomly selected 100 real estate loans from the portfolio of all loans and found that 80 of these loans had balances below $170,000. The senior loan officer is concerned that the sample selected by the auditors is not representative of the overall portfolio. In particular, he is interested in knowing the expected proportion of loans in the portfolio that would have balances below $170,000. You are asked to conduct an appropriate analysis and write a short report to the senior loan officers with your conclusion about the sample.

6-24. MP-3 players, and most notably the Apple iPod, have become an industry standard for people who want to have access to their favorite music and videos in a portable environment. The iPod can store massive numbers of songs and videos with its 60-GB hard drive. Although owners of the iPod have the potential to store lots of data, a recent study showed that the actual disk storage being used is normally distributed with a mean equal to 1.95 GB and a standard deviation of 0.48 GB. Suppose a competitor to Apple is thinking of entering the market with a low-cost iPod clone that

FIGURE 6.5

Probabilities from the Normal Curve for Fairfax Real Estate



positive $z = 0.50$. The probability in the table for $z = 0.50$ corresponds to the probability of a z-value occurring between $z = 0.50$ and $z = 0.0$. This is the same as the probability of a z-value falling between $z = -0.50$ and $z = 0.00$. Thus, from the standard normal table (Table 6.1 or Appendix D), we get

$$P(-0.50 \leq z \leq 0.00) = 0.1915$$

This is the area between $x = 14$ and $\mu = 16$ in Figure 6.5. We now add 0.1915 to 0.5000, which is the probability of a value exceeding $\mu = 16$. Therefore, the probability that a home will require 14 or more days to sell is

$$P(x \geq 14) = 0.1915 + 0.5000 = 0.6915$$

This is illustrated in Figure 6.5. Thus, there is nearly a 70% chance that a home will require at least 14 days to sell.

Business Application

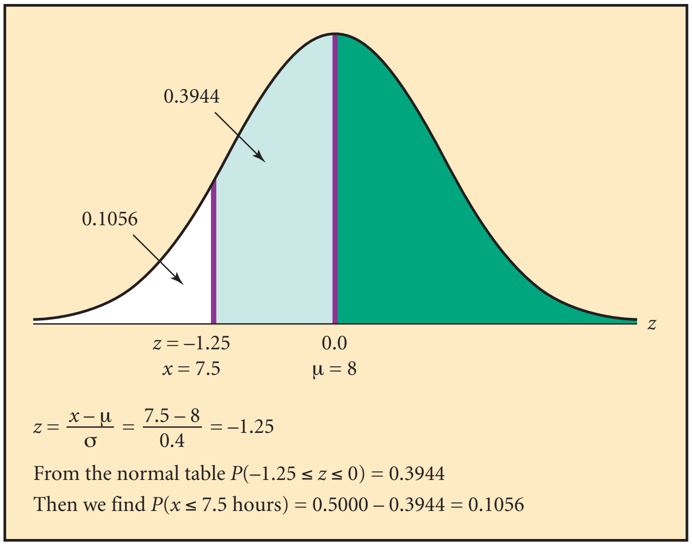
LONGLIFE BATTERY COMPANY Several states, including California, have passed legislation requiring automakers to sell a certain percentage of zero-emissions cars within their borders. One current alternative is battery-powered cars. The major problem with battery-operated cars is the limited time they can be driven before the batteries must be recharged. Longlife Battery, a start-up company, has developed a battery pack it claims will power a car at a sustained speed of 45 miles per hour for an average of 8 hours. But of course there will be variations: Some battery packs will last longer and some shorter than 8 hours. Current data indicate that the standard deviation of battery operation time before a charge is needed is 0.4 hours. Data show a normal distribution of uptime on these battery packs. Automakers are concerned that batteries may run short. For example, drivers might find an “8-hour” battery that lasts 7.5 hours or less unacceptable. What are the chances of this happening with the Longlife battery pack?

To calculate the probability the batteries will last 7.5 hours or less, find the appropriate area under the normal curve shown in Figure 6.6. There is approximately 1 chance in 10 that a battery will last 7.5 hours or less when the vehicle is driven at 45 miles per hour.

Suppose this level of reliability is unacceptable to the automakers. Instead of a 10% chance of an “8-hour” battery lasting 7.5 hours or less, the automakers will accept no more than a 2% chance. Longlife Battery asks the question, what would the mean uptime have to be to meet the 2% requirement?

FIGURE 6.6

Longlife Battery Company



Assuming that uptime is normally distributed, we can answer this question by using the standard normal distribution. However, instead of using the standard normal table to find a probability, we use it in reverse to find the z-value that corresponds to a known probability. Figure 6.7 shows the uptime distribution for the battery packs. Note, the 2% probability is shown in the left tail of the distribution. This is the allowable chance of a battery lasting 7.5 hours or less. We must solve for μ, the mean uptime that will meet this requirement.

1. Go to the body of the standard normal table, where the probabilities are located, and find the probability as close to 0.48 as possible. This is 0.4798.
2. Determine the z-value associated with 0.4798. This is $z = 2.05$. Because we are below the mean, the z is negative. Thus, $z = -2.05$.
3. The formula for z is

$$z = \frac{x - \mu}{\sigma}$$

4. Substituting the known values, we get

$$-2.05 = \frac{7.5 - \mu}{0.4}$$

FIGURE 6.7

Longlife Battery Company, Solving for the Mean

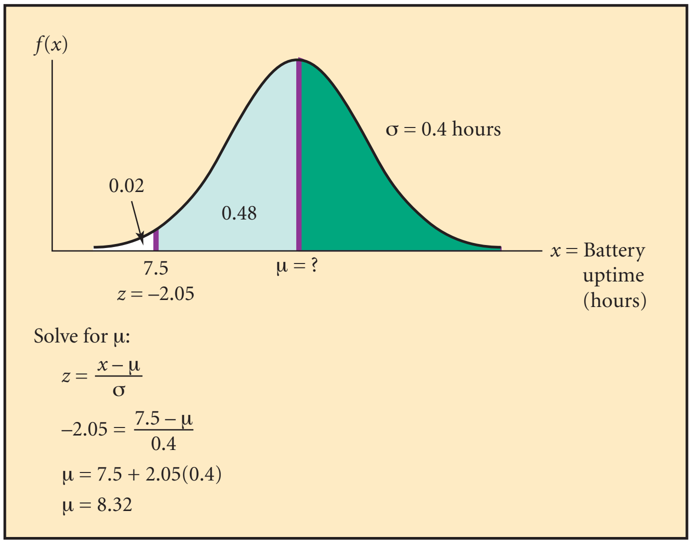
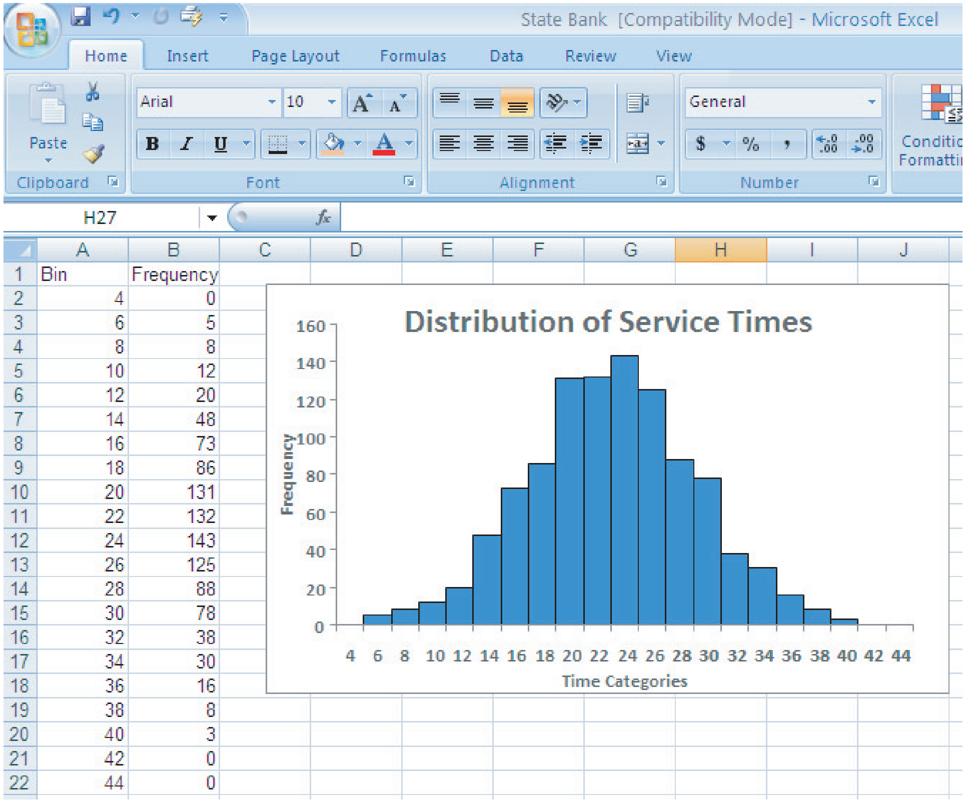


FIGURE 6.8

Excel 2007 Output for State Bank and Trust Service Times



Bin	Frequency
4	0
6	5
8	8
10	12
12	20
14	48
16	73
18	86
20	131
22	132
24	143
26	125
28	88
30	78
32	38
34	30
36	16
38	8
40	3
42	0
44	0

Excel 2007 Instructions:

1. Open file: State Bank.xls.
2. Create bins (upper limit of each class).
3. Select **Data > Data Analysis**.
4. Select **Histogram**.
5. Define data and bin ranges.
6. Check **Chart Output**.
7. Define Output Location.

Minitab Instructions (for similar results):

1. Open file: State Bank. MTW.
2. Choose **Graph > Histogram**.
3. Click **Simple**.
4. Click **OK**.
5. In **Graph Variables**, enter data column **Service Time**.
6. Click **OK**.