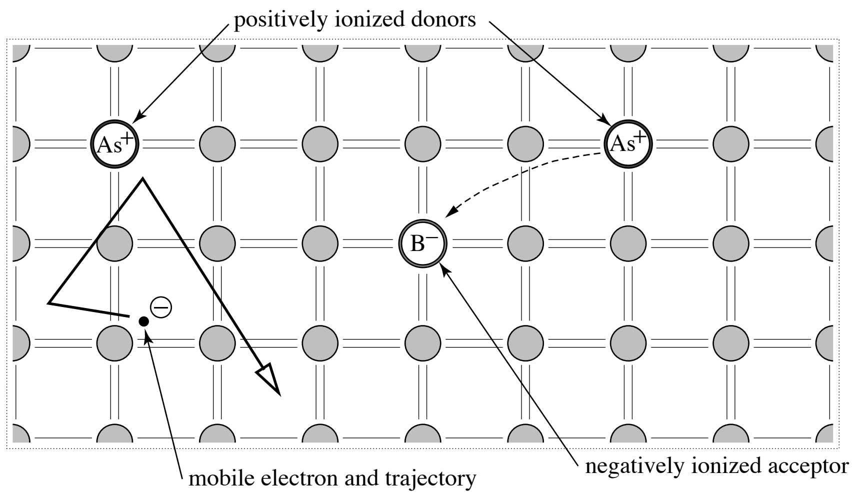
Compensation

Example shows $N_d > N_a$



- Applying charge neutrality with four types of charged species:

$$\rho = -qn_o + qp_o + qN_d - qN_a = q(p_o - n_o + N_d - N_a) = 0$$

we can substitute from the mass-action law $n_o p_o = n_i^2$ for either the electron concentration or for the hole concentration: which one is the majority carrier?

answer (not surprising): $N_d > N_a$ --> electrons

$N_a > N_d$ --> holes

Carrier Concentrations in Compensated Silicon

- For the case where $N_d > N_a$, the electron and hole concentrations are:

$$n_o \cong N_d - N_a \quad \text{and} \quad p_o \cong \frac{n_i^2}{N_d - N_a}$$

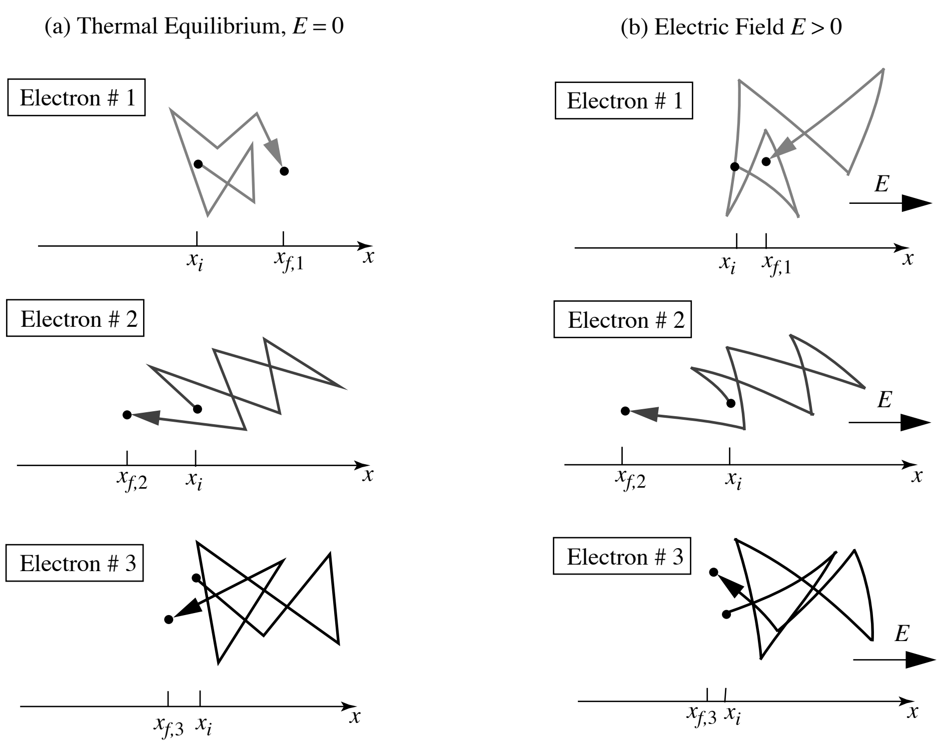
- For the case where $N_a > N_d$, the hole and electron concentrations are:

$$p_o \cong N_a - N_d \quad \text{and} \quad n_o \cong \frac{n_i^2}{N_a - N_d}$$

Note that these approximations assume that $| N_d - N_a | \gg n_i$, which is nearly always true.

Carrier Transport: Drift

- If an electric field is applied to silicon, the holes and the electrons "feel" an electrostatic force $F_e = (+q \text{ or } - q)E$.
- Picture of effect of electric field on representative electrons: moving at the thermal velocity = 10^7 cm/s ... *very fast*, but colliding every 0.1 ps = 10^{-13} s. Distance between collsions = 10^7 cm/s x 10^{-13} cm = 0.01 μm



* x_i = initial position   * $x_{f,\,n}$ = final position of electron *n* after 7 collisions

- The average of the position changes for the case with $E > 0$ is $\overline{\Delta x} < 0$

Drift Velocity and Mobility

- The *drift velocity* v_{dn} of electrons is defined as:

$$v_{dn} = \frac{\overline{\Delta x}}{\Delta t}$$

- Experiment shows that the drift velocity is proportional to the electric field for electrons

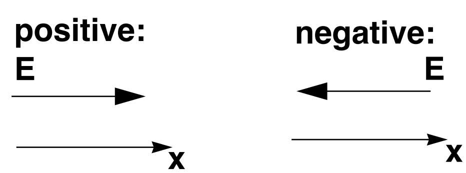
$$v_{dn} = -\mu_n E,$$

with the constant μ_n defined as the *electron mobility.*

- Holes drift in the direction of the applied electric field, with the constant μ_p defined as the *hole mobility.*

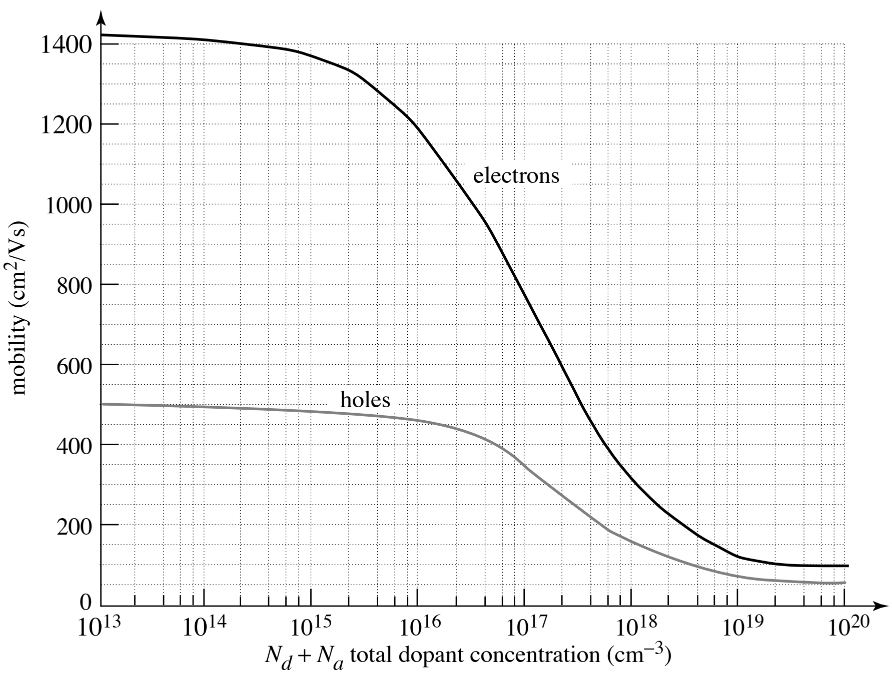
$$v_{dp} = \mu_p E$$

How do we know what's positive and what's negative?



Electron and Hole Mobilities

- mobilities vary with doping level -- plot is for 300 K = room temp.



- "typical values" for bulk silicon - assuming around 5×10^{16} cm^{-3} doping

$$\mu_n = 1000\ \text{cm}^2/(\text{Vs})$$

$$\mu_p = 400\ \text{cm}^2/(\text{Vs})$$

- at electric fields greater than around 10^4 V/cm, the drift velocities saturate --> max. out at around 10^7 cm/s. Velocity saturation is very common in VLSI devices, due to sub-micron dimensions

Carrier Transport: Drift Current Density

Electrons drifting opposite to the electric field are carrying negative charge; therefore, the *drift current density* is:

$$J_n^{dr} = (-q)\, n\, v_{dn} \qquad \text{units: } \mathrm{Ccm^{-2}\, s^{-1} = Acm^{-2}}$$

$$\boxed{J_n^{dr} = (-q)\, n\, (-\mu_n E) = q\, n\, \mu_n E}$$

Note that J_n^{dr} is in the *same* direction as the electric field.

For holes, the mobility is μ_p and the drift velocity is in the same direction as the electric field: $v_{dp} = \mu_p E$

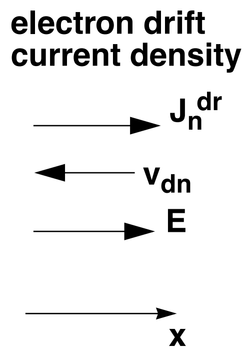
The hole drift current density is:

$$J_p^{dr} = (+q)\, p\, v_{dp}$$

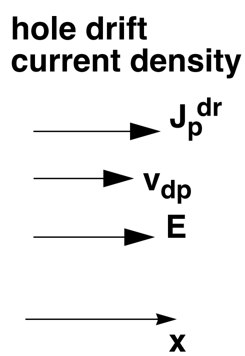
$$\boxed{J_p^{dr} = q\, p\, \mu_p E}$$

Drift Current Directions and Signs

- For electrons, an electric field in the +*x* direction will lead to a drift velocity in the -*x* direction ($v_{dn} < 0$) and a drift current density in the +*x* direction ($J_n^{dr} > 0$).



- For holes, an electric field in the +*x* direction will lead to a drift velocity in the +*x* direction ($v_{dp} > 0$) and a drift current density in the +*x* direction ($J_n^{dr} > 0$).



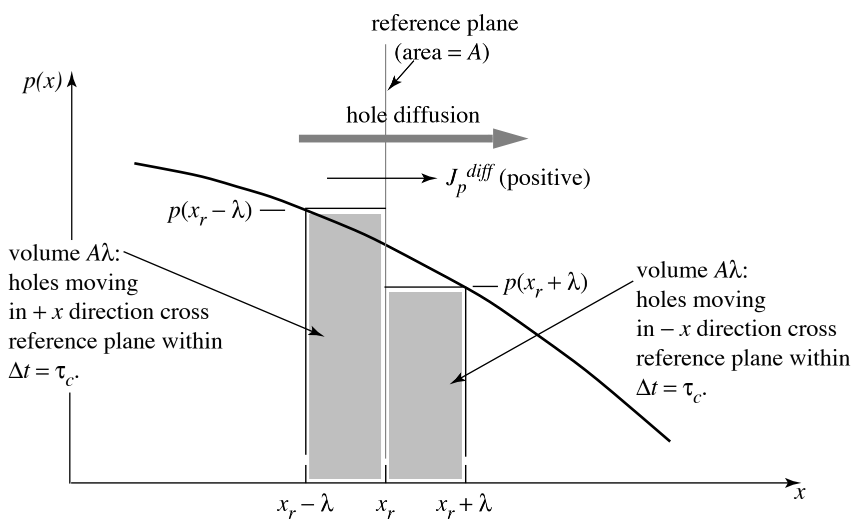
Carrier Transport: Diffusion

Diffusion is a transport process driven by gradients in the concentration of particles in random motion and undergoing frequent collisions -- such as ink molecules in water ... or holes and electrons in silicon.

Mathematics: find the number of carriers in a volume $A\lambda$ on either side of the reference plane, where λ is the mean free path between collisions.

- Some numbers: average carrier velocity = $v_{th} = 10^7$ cm/s, average interval between collisions = $\tau_c = 10^{-13}$ s = 0.1 picoseconds

  mean free path = $\lambda = v_{th}\,\tau_c = 10^{-6}$ cm = 0.01 μm



- half of the carriers in each volume will pass through the plane before their next collision, since their motion is random

Carrier Transport:

Diffusion Current Density

- Current density = (charge) x (# carriers per second per area):

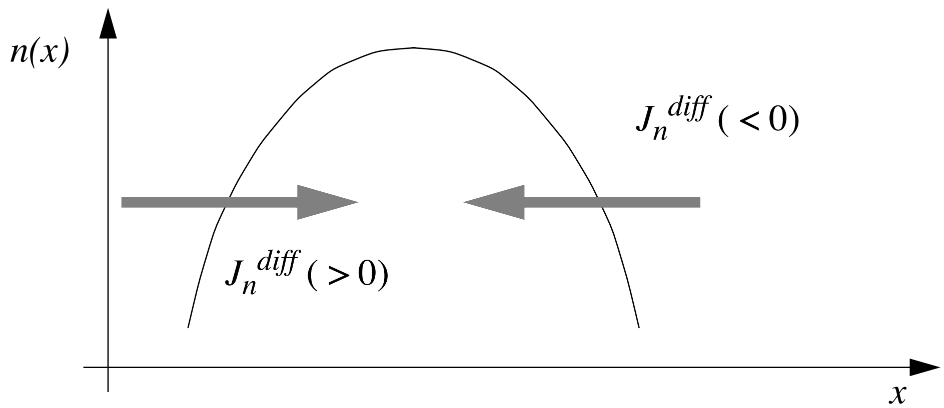
$$J_p^{diff} = q\left[\frac{\frac{1}{2}p(x-\lambda)A\lambda - \frac{1}{2}p(x+\lambda)A\lambda}{A\tau_c}\right]$$

- If we assume that λ is much smaller than the dimensions of our device, then we can consider $\lambda = dx$ and use Taylor expansions :

$$J_p^{diff} = -qD_p\frac{dp}{dx}$$, where $D_p = \lambda^2 / \tau_c$ is the diffusion coefficent

Carrier Transport by Diffusion (cont.)

- Electrons diffuse down the concentration gradient, yet carry negative charge --> electron diffusion current density points in the direction of the gradient



- Total current density: add drift and diffusion components for electrons and for holes --

$$J_n = J_n^{dr} + J_n^{diff} = qn\mu_n E + qD_n \frac{dn}{dx}$$

$$J_p = J_p^{dr} + J_p^{diff} = qp\mu_p E - qD_p \frac{dp}{dx}$$

- Fortunately, we will be able to eliminate one or the other component in finding the internal currents in microelectronic devices.