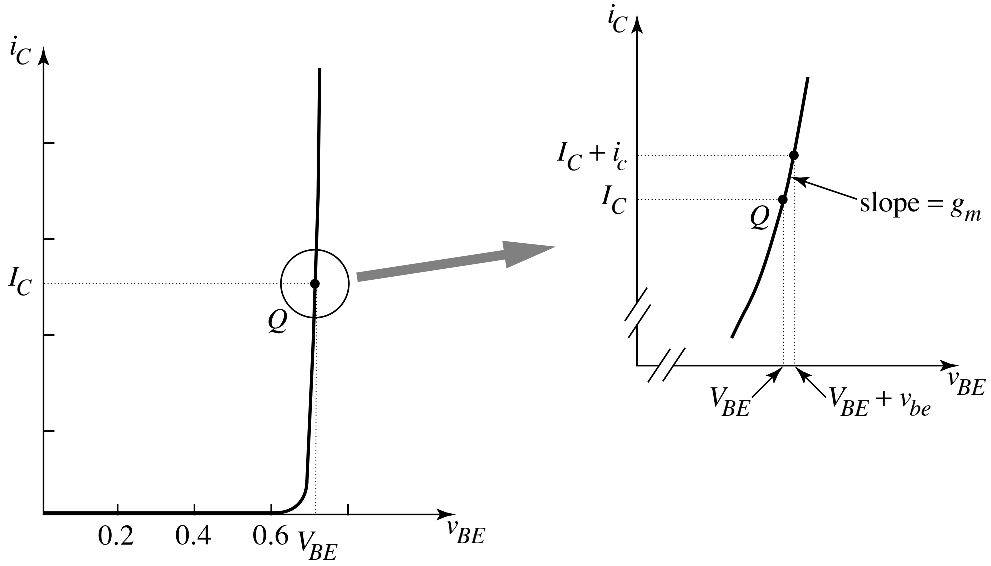
Small-Signal Model
of the Forward-Active npn BJT

- Transconductance (same concept as for MOSFET):

$$g_m = \left.\frac{\partial i_C}{\partial v_{BE}}\right|_Q$$

Ebers-Moll (forward-active): $i_C = I_S e^{v_{BE}/V_{th}}$



Evaluating the derivative, we find that

$$g_m = \left(\frac{I_S}{V_{th}}\right) e^{V_{BE}/V_{th}} = \frac{I_C}{V_{th}}$$

Input Resistance

- The collector current is a function of the base current in the forward-active region (recall $I_C = \beta_F I_B$). At the operating point Q, we define

$$\beta_o = \left.\frac{\partial i_C}{\partial i_B}\right|_Q$$

  and so $i_c = \beta_o i_b$. (Note that the "DC beta" β_F and the small-signal β_o are both highly variable from device to device)

- Since the base current is therefore a function of the base-emitter voltage, we define the input resistance r_π as:

$$r_\pi^{-1} = \left.\frac{\partial i_B}{\partial v_{BE}}\right|_Q = \left.\frac{\partial i_B}{\partial i_C}\right|_Q \left.\frac{\partial i_C}{\partial v_{BE}}\right|_Q = \left(\frac{1}{\beta_o}\right) g_m$$

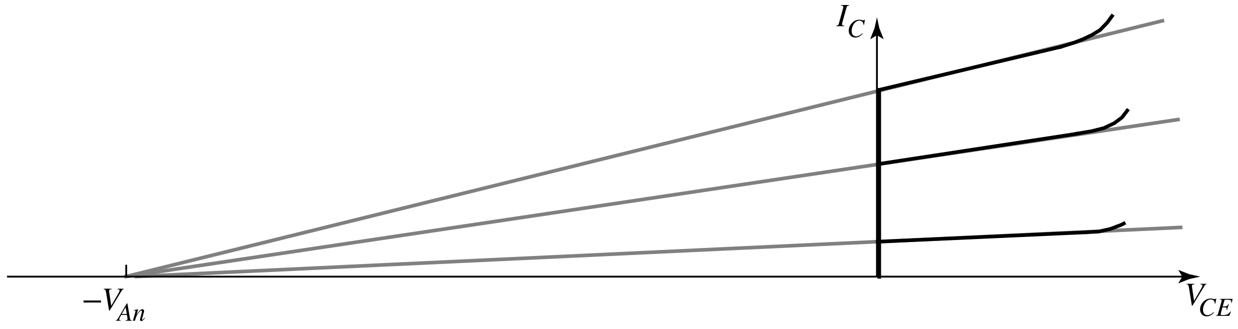
- Solving for the input resistance

$$\boxed{r_\pi = \frac{\beta_o}{g_m} = \frac{\beta_o V_{th}}{I_C} = \frac{kT\beta_o}{qI_C}}$$

- For a high input resistance (often desirable), we need a high current gain or a low DC bias current.

Output Resistance

- The Ebers-Moll model has perfect current source behavior in the forward-active region -- actual characteristics show some increase:



- Why? Base width shrinks due to encroachment by base-collector depletion region

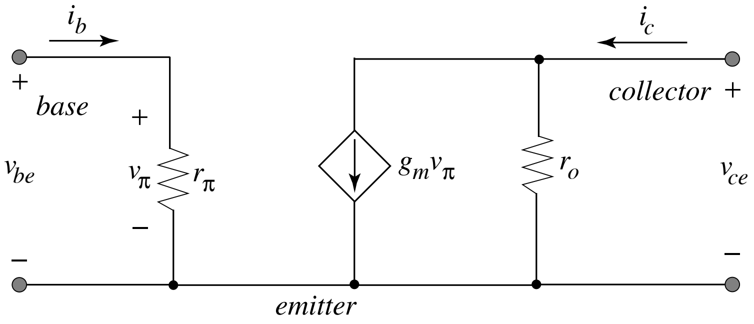
  Approximate model: introduce Early voltage V_{An} to model increase in i_C

  Model: $$i_C = I_S e^{v_{BE}/V_{th}}\left(1 + \frac{v_{CE}}{V_{An}}\right)$$

- Output resistance:

$$r_o^{-1} = \left.\frac{\partial i_C}{\partial v_{CE}}\right|_Q \cong \frac{I_C}{V_{An}}$$

Numerical Values of Small-Signal Elements



- Transconductance:

$I_C = 100\ \mu A$, $V_{th} = 25$ mV --> $\boxed{g_m = 4\ \text{mS} = 4 \times 10^{-3}\ \text{S}}$

Note: g_m varies *linearly* with collector current and is independent of device geometry, in contrast to the MOSFET

- Input resistance:

$\beta_o = 100$, $I_C = 100\ \mu A$, $V_{th} = 25$ mV --> $\boxed{r_\pi = 25\ k\Omega}$

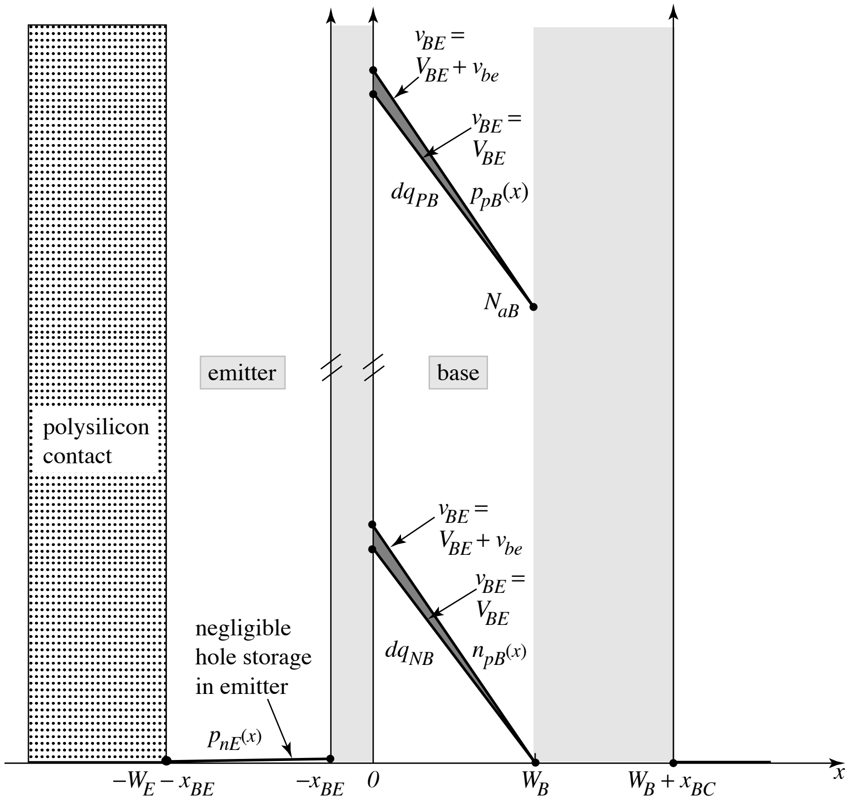
- Output resistance:

$I_C = 100\ \mu A$, $V_{An} = 35$ V --> $\boxed{r_o = 350\ k\Omega}$

V_{An} = Early voltage increases with increasing base width and decreases with decreasing base doping.

Charge-Storage Elements: Base-Charging Capacitance C_b

- Minority electrons are stored in the base -- this charge q_{NB} is a function of the base-emitter voltage



- base is still neutral... majority carriers neutralize the injected electrons
  $q_{PB} = q_{NB}$

$$C_b = \left.\frac{\partial q_{PB}}{\partial v_{BE}}\right|_Q$$

Base Transit Time

- The electron charge in the base is found by integrating the electron concentration in the base -- the area is A_E (under the emitter):

$$q_{PB} = -q_{NB} = -\int_0^{W_B} -qA_E n_{pB}(x)dx = \frac{1}{2} qA_E W_B n_{pBo} e^{v_{BE}/V_{th}}$$

- The stored charge is proportional to the collector current:

$$q_{PB} = \frac{1}{2} W_B (W_B / D_{nB}) \left(\frac{qA_E D_{nB}}{W_B} \right) n_{pBo} e^{v_{BE}/V_{th}} = \left(\frac{W_B^2}{2D_{nB}} \right) i_C$$

- The proportionality constant looks like a diffusion time (it is) and is defined as the base transit time:

$$\tau_F = \frac{W_B^2}{2D_{nB}}$$

A typical transit time is τ_F = 10 ps for an oxide-isolated npn BJT.

- The base-charging capacitance is:

$$\boxed{C_b = \left.\frac{\partial q_{PB}}{\partial v_{BE}}\right|_Q = g_m \tau_F}$$

Complete Small-Signal Model

- Add the depletion capacitance from the base-emitter junction to find the total base-emitter capacitance: $C_\pi = C_{jE} + C_b$

$$C_{jE} = \sqrt{2}C_{jEo}$$

C_{jEo} is proportional to the emitter-base junction area (A_E)

- Depletion capacitance from the base-collector junction: C_μ

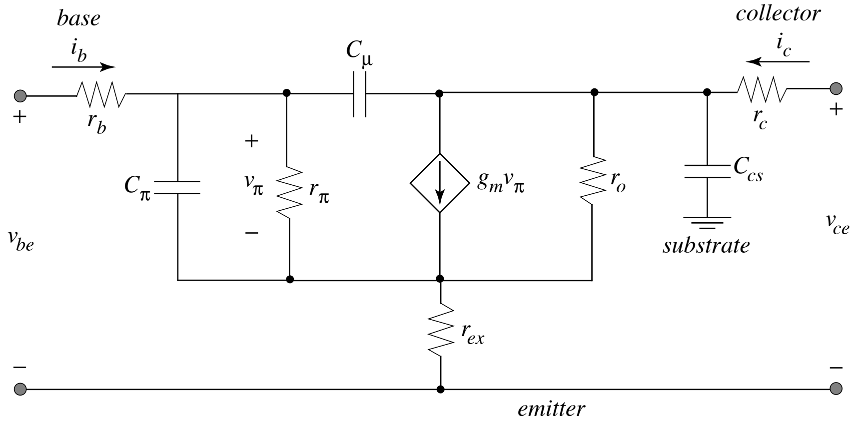
$$C_\mu = \frac{C_{\mu o}}{\sqrt{1 + V_{CB}/\phi_{Bc}}}$$

$C_{\mu o}$ is proportional to the base-collector junction area (A_C)

- Depletion capacitance from collector (n^+ buried layer) to bulk: C_{cs}

$$C_{cs} = \frac{C_{cso}}{\sqrt{1 + V_{CS}/\phi_{Bs}}}$$

C_{cso} is proportional to the collector-substrate junction area (A_S)



npn BJT SPICE model

Close correspondence to Ebers-Moll and small-signal models

Name	Parameter Description	Units
IS	transport saturation current [I_S]	Amps
BF	ideal maximum forward beta [β_F]	None
VAF	forward Early voltage [V_{An}]	Volts
BR	ideal maximum reverse beta [β_R]	None
RB	zero bias base resistance [r_b]	Ohms
RE	emitter resistance [r_{ex}]	Ohms
RC	collector resistance [r_c]	Ohms
CJE	B-E zero-bias depletion capacitance [C_{jEo}]	Farads
VJE	B-E built-in potential [ϕ_{Be}]	Volts
MJE	B-E junction exponential factor	None
CJC	B-C zero-bias depletion capacitance [$C_{\mu o}$]	Farads
VJC	B-C built-in potential [ϕ_{Bc}]	Volts
MJC	B-C junction exponential factor	None
CJS	substrate zero-bias depletion capacitance [C_{cso}]	Farads
VJS	substrate built-in potential [ϕ_{Bs}]	Volts
MJS	substrate junction exponential factor	None
TF	ideal forward transit time [τ_F]	Seconds

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.MODEL MODQN NPN IS=1E-17 BF=100 VAF=25 TF=50P
+ CJE=8E-15 VJE=0.95 MJE=0.5 CJC=22E-15 VJC=0.79 MJC=0.5
+ CJS=41E-15 VJS=0.71 MJS=0.5 RB=250 RC=200 RE=5
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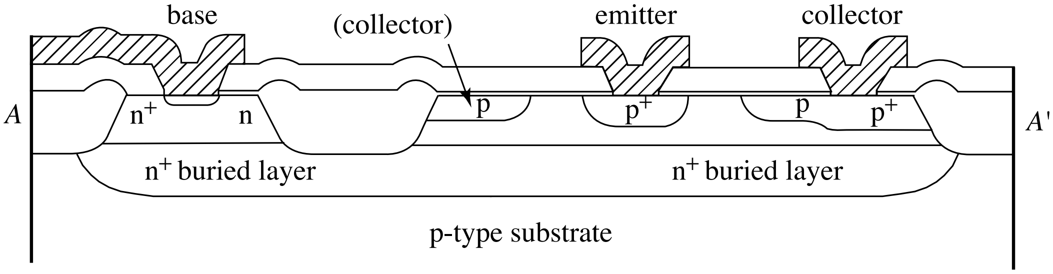
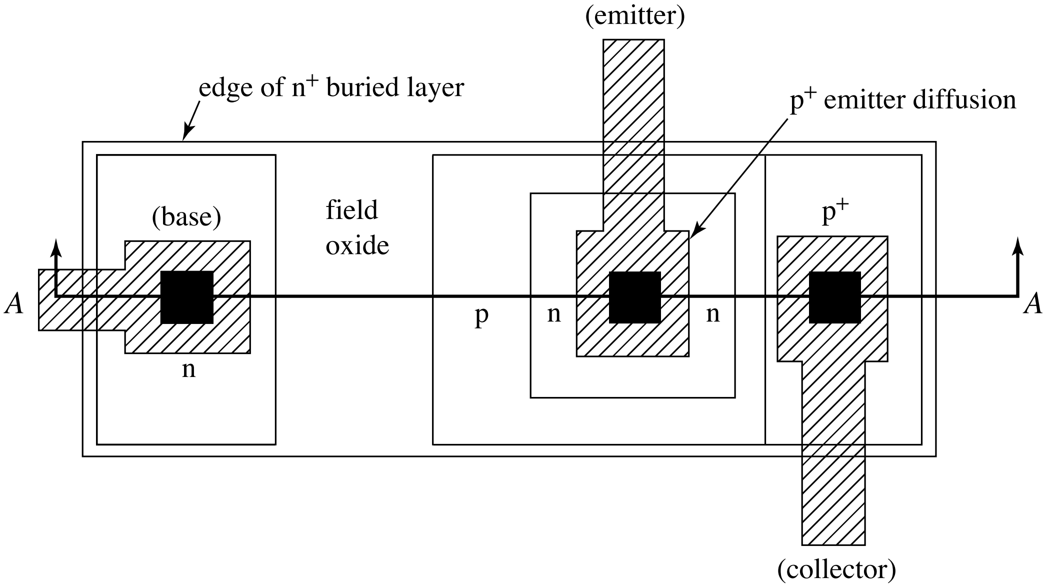
The Lateral pnp BJT

- vertical pnp transistors cannot be made in the fabrication sequence that makes the npn oxide-isolated transistor.
- a “free” pnp *can* be made in which holes are injected *laterally* at the perimeter of a p^+ region and then diffuse across an n-type base region,where they are collected by another p region



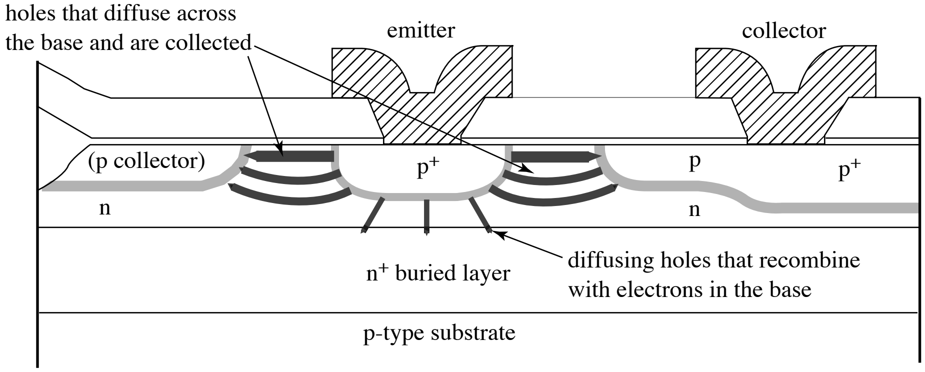
(a)



(b)

Circuit models for the Lateral pnp

- Hole flux in the base for the lateral pnp



- In the forward-active region, the collector current -i_C is a function of v_{EB} and the emitter-collector voltage v_{EC}.

$$-i_C = I_S e^{v_{EB}/V_{th}} \left(1 + \frac{v_{EC}}{V_{Ap}}\right)$$

- The current gain is inferior: β_F = 30-50; the base-transit time is more than an order of magnitude longer ... τ_F = 500-700 ps