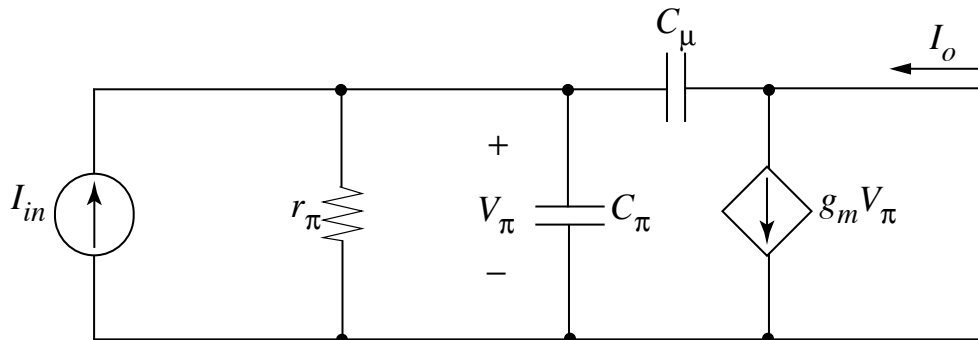


Frequency Response of Transistor Amplifiers

- Simplest case: CE short-circuit current gain $A_i(j\omega)$ as a function of frequency



Kirchhoff's current law at the output node:

$$I_o = g_m V_\pi - V_\pi j\omega C_\mu$$

Kirchhoff's current law at the input node:

$$I_s = \frac{V_\pi}{Z_\pi} + V_\pi j\omega C_\mu \quad \text{where} \quad Z_\pi = r_\pi \parallel \left(\frac{1}{j\omega C_\pi} \right)$$

- Solving for V_π at the input node:

$$V_\pi = \frac{I_s}{(1/Z_\pi) + j\omega C_\mu}$$

Short-Circuit Gain Frequency Response

- Substituting V_π into the output node equation--

$$\frac{I_o}{I_s} = \frac{g_m Z_\pi \left(1 - \frac{j\omega C_\mu}{g_m}\right)}{1 + j\omega C_\mu Z_\pi}$$

- Substituting for Z_π and simplifying --

$$\frac{I_o}{I_s} = \frac{g_m r_\pi \left(1 - \frac{j\omega C_\mu}{g_m}\right)}{1 + j\omega r_\pi (C_\pi + C_\mu)} = \frac{\beta_o \left(1 - \frac{j\omega C_\mu}{g_m}\right)}{1 + j\omega r_\pi (C_\pi + C_\mu)} = \beta_o \left[\frac{1 - j\frac{\omega}{\omega_z}}{1 + j\frac{\omega}{\omega_p}} \right]$$

Current gain has one pole:

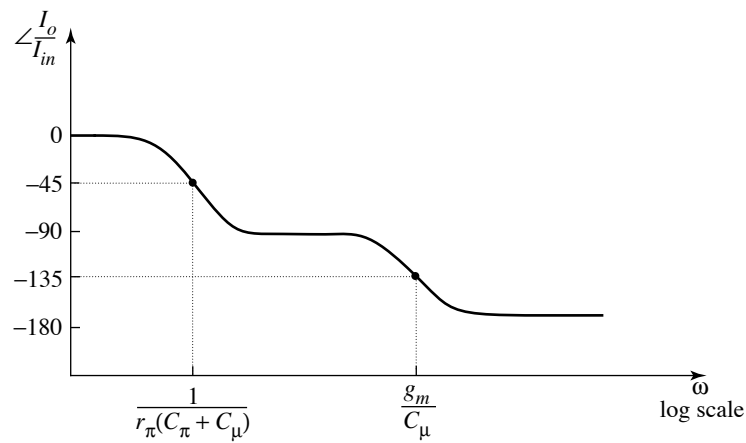
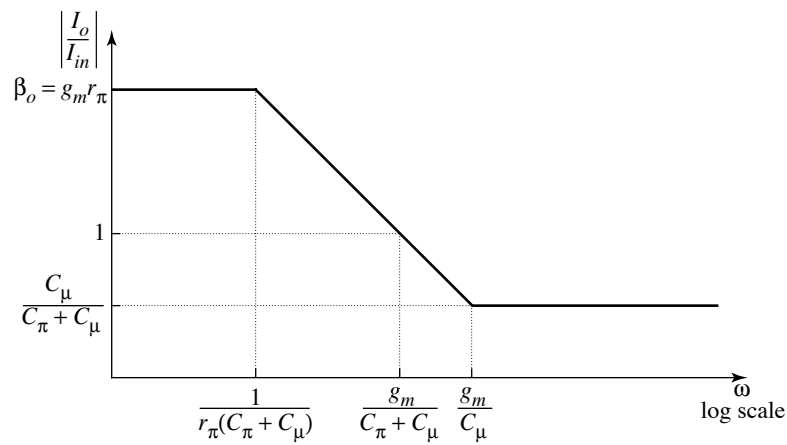
$$\omega_p = (r_\pi (C_\pi + C_\mu))^{-1}$$

and one zero

$$\omega_z = (g_m^{-1} C_\mu)^{-1} \gg \omega_p$$

Bode Plot of Short-Circuit Current Gain

- Note low frequency magnitude of gain is β_o



- Frequency at which current gain is reduced to 0 dB is defined as the **transition frequency** ω_T . Neglecting the zero,

$$\omega_T = \frac{g_m}{(C_\pi + C_\mu)}$$

Transition Frequency of the Bipolar Transistor

- Dependence of transition time $\tau_T = \omega_T^{-1}$ on the bias collector current I_C :

$$\tau_T = \frac{1}{\omega_T} = \frac{C_\pi + C_\mu}{g_m} = \frac{g_m \tau_F + C_{jE} + C_\mu}{g_m}$$

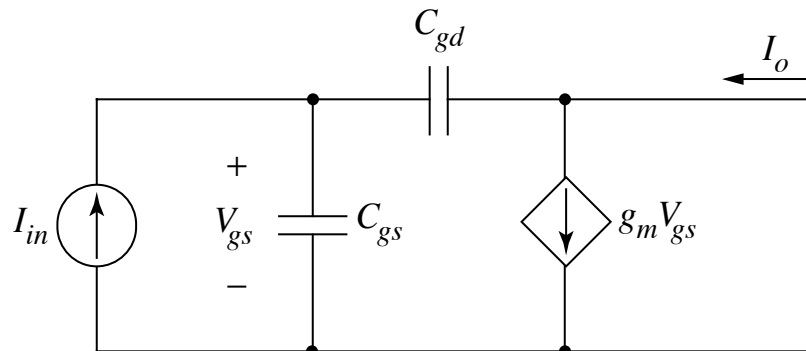
$$\tau_T = \tau_F + \left(\frac{C_{jE} + C_\mu}{g_m} \right) = \tau_F + \frac{V_{th}}{I_C} (C_{jE} + C_\mu)$$

- If the collector current is increased enough to make the second term negligible, then the minimum τ_T is the base transit time, τ_F . In practice, the ω_T decreases at very high values of I_C due to other effects and the minimum τ_T may not be achieved.
- Numerical values of $f_T = (1/2\pi)\omega_T$ range from 10 MHz for lateral pnp's to 10 GHz for oxide-isolated npn's

Note that the small-signal model is not valid above f_T (due to distributed effects in the base) and the zero in the current gain is not observed

Common-Source Current Gain

- CS amplifier has a non-infinite input *impedance* for $\omega > 0$ and we can measure its small-signal current gain.



- Analysis is similar to CE case; result is

$$\frac{I_o}{I_{in}} = \frac{g_m \left(1 - \frac{j\omega C_{gd}}{g_m} \right)}{j\omega(C_{gs} + C_{gd})} \approx \frac{g_m}{\omega(C_{gs} + C_{gd})}$$

- Transition frequency for the MOSFET is

$$\omega_T \approx \frac{g_m}{C_{gs} + C_{gd}}$$

Transition Frequency of the MOSFET

- Substitution of gate-source capacitance and transconductance:

$$C_{gs} = \frac{2}{3}WLC_{ox} \gg C_{gd} \text{ and } g_m = \frac{W}{L}\mu_n C_{ox}(V_{GS} - V_{Tn})$$
$$\omega_T \approx \frac{g_m}{C_{gs}} = \frac{\frac{W}{L}\mu_n C_{ox}(V_{GS} - V_{Tn})}{\frac{2}{3}WLC_{ox}} = \frac{3}{2}\mu_n \left[\frac{(V_{GS} - V_{Tn})}{L} \right] L$$

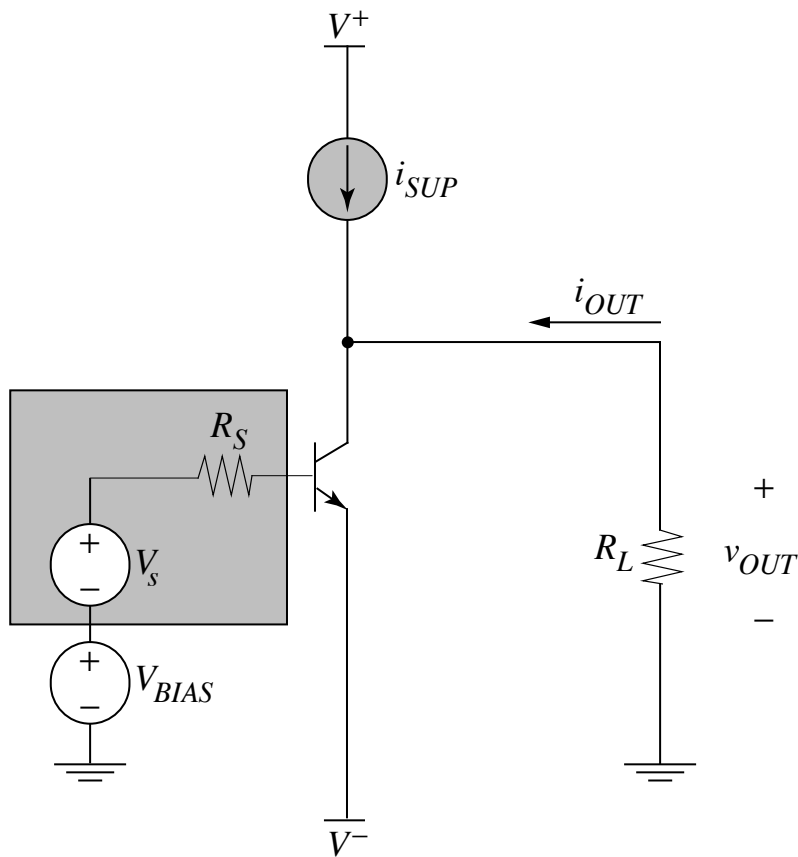
- The transition time is the inverse of ω_T and can be written as the average time for electrons to drift from source to drain

$$\tau_T = \frac{L}{\mu_n \left[\frac{2(V_{GS} - V_{Tn})}{3} \right]} = \frac{L}{|v_{dr}|}$$

velocity saturation causes τ_T to decrease linearly with L ; however, submicron MOSFETs have transition frequencies that are approaching those for oxide-isolated BJTs

Frequency Response of Voltage Amplifiers

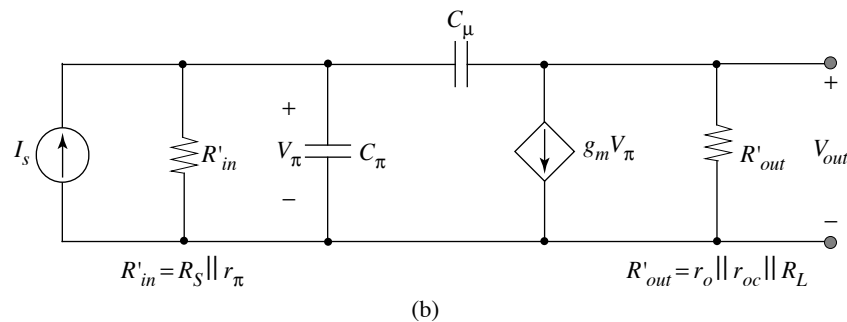
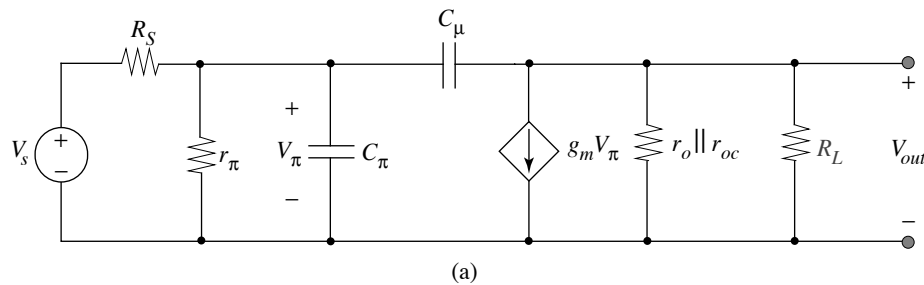
- Common-emitter amplifier:



Procedure: substitute small-signal model and perform phasor analysis

Brute Force Phasor Analysis

- “Exact” analysis: transform into Norton form at input to facilitate nodal analysis



Note that C_{cS} is omitted, along with r_b

Details: see Section 10.4

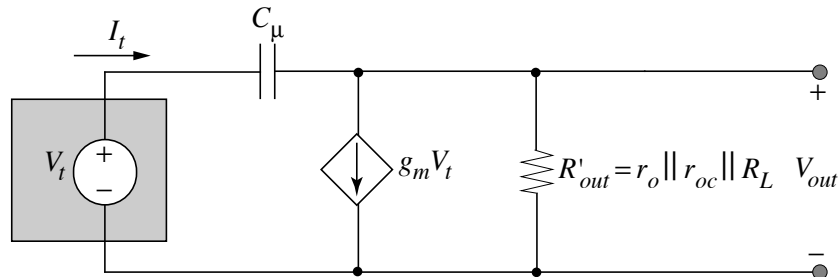
Factor (approximately!) into one high-frequency zero and two poles $\omega_1 \ll \omega_2$

$$\omega_1^{-1} = (r_\pi \parallel R_S)(C_\pi + (1 + g_m r_o \parallel r_{oc} \parallel R_L)C_\mu) + (r_o \parallel r_{oc} \parallel R_L)C_\mu$$

$$\omega_2^{-1} = \frac{(r_o \parallel r_{oc} \parallel R_L)(r_\pi \parallel R_S)C_\mu C_\pi}{(r_\pi \parallel R_S)(C_\pi + (1 + g_m r_o \parallel r_{oc} \parallel R_L)C_\mu) + (r_o \parallel r_{oc} \parallel R_L)C_\mu}$$

The Miller Approximation

- The “exact” analysis is not particularly helpful for gaining insight into the frequency response ... consider the effect of C_μ on the input only



neglect the feedforward current I_μ in comparison with $g_m V_\pi$... a good approximation

$$I_t = (V_t - V_o) / Z_\mu$$

$$V_o = -g_m V_t R_L / (R_L + R'_{out}) = A_{vC_\mu} V_t$$

where A_{vC_μ} is the low frequency voltage gain across C_μ

$$I_t = V_t (1 - A_v) / Z_\mu$$

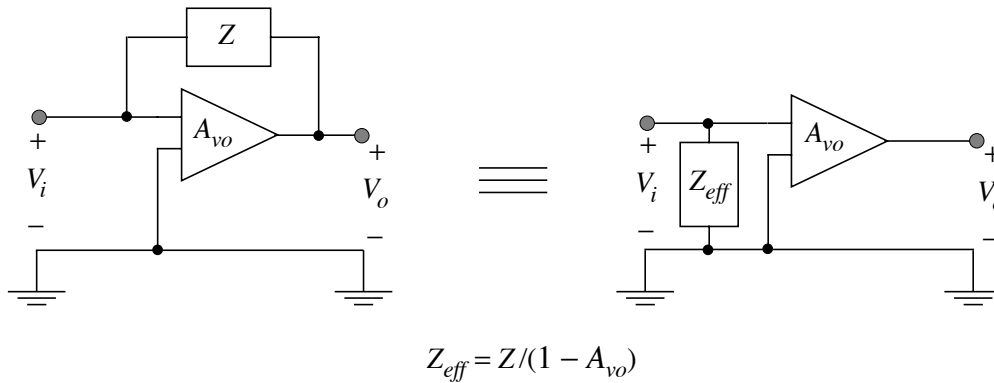
$$Z_{eff} = V_t / I_t = Z_\mu / (1 - A_v)$$

$$Z_{eff} = \frac{1}{j\omega C_\mu \left(\frac{1}{1 - A_{vC_\mu}} \right)} = \frac{1}{j\omega (C_\mu (1 - A_{vC_\mu}))} = \frac{1}{j\omega C_M}$$

$C_M = (1 - A_{vC_\mu}) C_\mu$ is the **Miller capacitor**

Generalized Miller Approximation

- An impedance Z connected across an amplifier with voltage gain A_{vZ} can be replaced by an impedance to ground ... multiplied by $(1-A_{vZ})$



- Common-emitter and common-source:

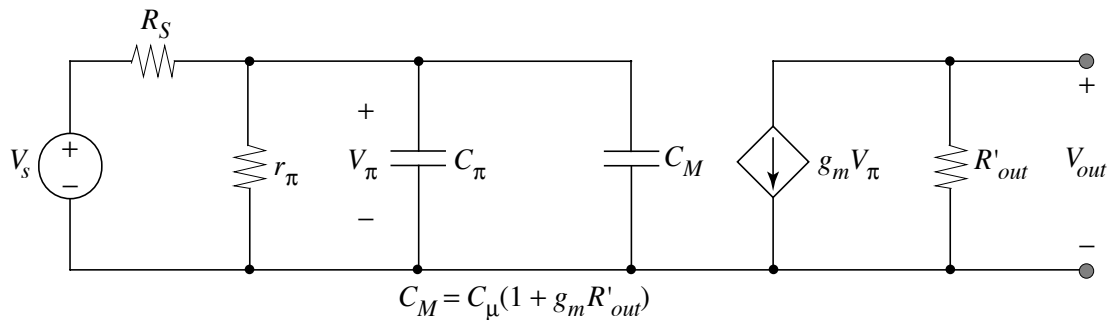
$A_{vZ} =$ large and negative for C_{μ} or C_{gd} --> capacitance at the input is **magnified**

- Common-collector and common-drain:

$A_{vZ} \approx 1$ --> capacitance at the input due to C_{π} or C_{gs} is greatly **reduced**

Voltage Gain vs. Frequency for CE Amplifier Using the Miller Approximation

- The Miller capacitance is lumped together with C_π , which results in a single-pole low-pass RC filter at the input



Transfer function has one pole and no zero after Miller approximation:

$$\omega_{3dB}^{-1} = (r_\pi \parallel R_S)(C_\pi + C_M)$$

$$\omega_{3dB}^{-1} = (r_\pi \parallel R_S)[C_\pi + (1 + g_m r_o \parallel r_{oc} \parallel R_L)C_\mu]$$

$$\omega_{3dB}^{-1} \approx \omega_1^{-1} \text{ from the exact analysis (final term } R'_{out} C_\mu \text{ is missing)}$$

