

Ohm's Law for Silicon

Bulk silicon: uniform doping concentration, away from surfaces

n-type example: in equilibrium, $n_o = N_d$ and $\rho = 0$.

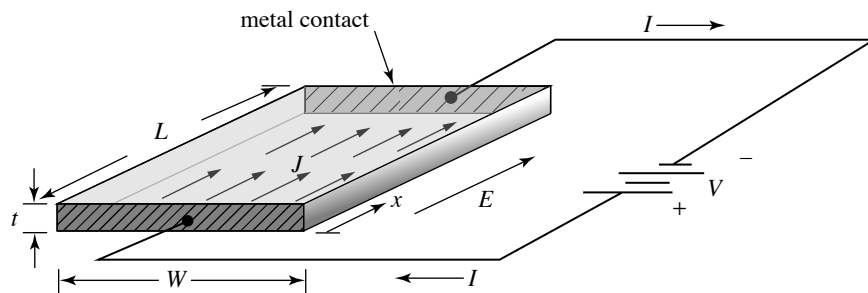
When we apply an electric field, $n = N_d$ and $\rho = 0 \dots n$

Currents in n-type bulk silicon with an applied electric field E :

$$J_n = qn\mu_n E + qD_n \frac{dn}{dx} \cong q\mu_n N_d E = \sigma_n E$$

where σ_n is the *conductivity* [Units: S/cm = 1 / (Ω cm)]

note: holes contribute almost nothing to the conductivity of n-type silicon.



Doped Silicon Resistors

* Find the current density in the resistor:

$$J_n = \sigma_n E = \sigma_n \frac{V_A}{L}$$

assumption: field is less than $E_{sat} = 10^4$ V/cm, so no velocity saturation

* Current is current density times cross sectional area:

$$I = \sigma_n \left(\frac{V_A}{L} \right) A = \left(\frac{\sigma_n A}{L} \right) V_A$$

$$R = \frac{V_A}{I} = \left(\frac{1}{\sigma_n} \right) \frac{L}{A} = \rho_n \frac{L}{A}$$

where ρ_n is the resistivity [units: Ω cm]

* *Silicon resistivities:*

500 Ω cm to 5 m Ω cm for doping concentrations from 10^{13} to 10^{19} cm $^{-3}$

Sheet Resistance

- Rewrite the resistance equation to separate (L / W) , the length-to-width ratio ... which is the number of “squares” N_{\square} from R_{\square} , the sheet resistance $= (\sigma_n t)^{-1}$

$$R = \frac{L}{q\mu_n N_d W t} = \left(\frac{1}{q\mu_n N_d t} \right) \frac{L}{W} = R_{\square} (L/W) = R_{\square} N_{\square}$$

The sheet resistance is under the control of the *process designer*; the number of squares is determined by the layout and is specified by the *IC designer*.

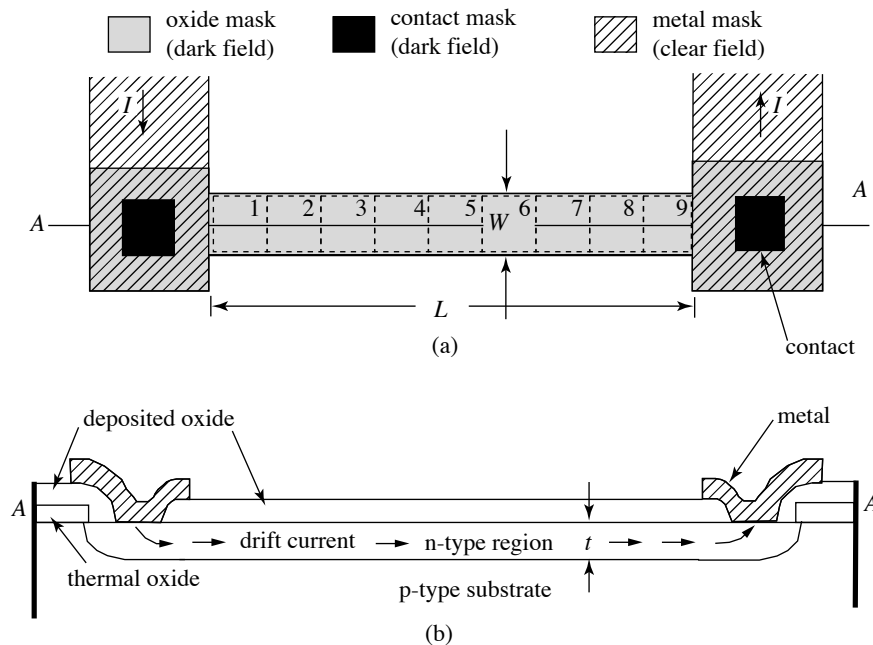
For average doping levels of 10^{15} cm^{-3} to 10^{19} cm^{-3} and a typical layer thickness of $0.5 \text{ }\mu\text{m}$, the sheet resistance ranges from $100 \text{ k}\Omega/\square$ to $10 \text{ }\Omega/\square$.

Other conducting materials: (MOSIS $1 \text{ }\mu\text{m}$ CMOS process)

	Ω / \square
n^+ polysilicon ($t = 500 \text{ nm}$)	20
aluminum ($t = 1 \text{ }\mu\text{m}$)	0.07
silicided polysilicon	5
silicided source/drain diffusion	3

Integrated Circuit Resistors

- Fabricate an n-type resistor in a p-type substrate using the process described in Chapter 2.



- Given the sheet resistance, we need to find the number of squares for this layout
 $L / W = 9$ squares

Laying Out a Resistor

- Rough approach:

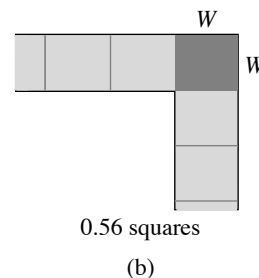
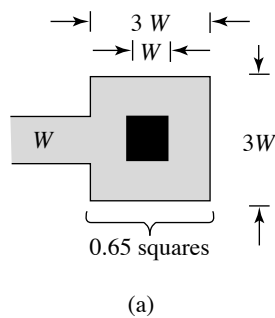
$$R \text{ known } \rightarrow N_{\square} = R / R_{\square}.$$

Select a width W (possibly the minimum to save area) \rightarrow the length $L = W N_{\square}$ and make a rectangle $L \times W$ in area

Add contact regions at the ends ... ignore their contribution to R

- More careful approach:

account for the contact regions and also, for corners



Measurement shows that the effective number of squares of the “dogbone” style contact region is 0.65 and for a 90° corner is 0.56.

For the resistor with $L / W = 9$, the contact regions add a significant amount to the total square count:

$$N_{\square} = 9 + 2 (0.65) = 10.3$$

In design, the contact regions and the corners should be accounted for to accurately determine the layout needed to yield the desired resistance.

Uncertainties in IC Fabrication

The precision of transistors and passive components fabricated using IC technology is surprisingly, *poor*!

Sources of variations:

- ion implant dose varies from point to point over the wafer and from wafer to wafer

- thicknesses of layers after annealing vary due to temperature variations across the wafer

- widths of regions vary systematically due to imperfect wafer flatness (leading to focus problems) and randomly due to raggedness in the photoresist edges after development

- etc., etc.

Quantifying Variations in Device Parameters

We will write an uncertain parameter (such as the acceptor conc.) as:

$$N_a = \bar{N}_a(1 \pm \varepsilon_{N_a})$$

where \bar{N}_a is the *average doping* and ε_{N_a} is the *normalized uncertainty*

(Note - we've swept a lot of probability and statistics under the rug here)

As an example, an average acceptor concentration $N_a = 10^{16} \text{ cm}^{-3}$ and a normalized uncertainty $\varepsilon_{N_a} = 0.04$ means that the acceptor concentration ranges from

$$0.96 \times 10^{16} \text{ cm}^{-3} \text{ to } 1.04 \times 10^{16} \text{ cm}^{-3}$$

How do variations combine to determine the variation in an IC resistor?

- assume that the variations are *independent*
- assume that the normalized uncertainty of a function of several variables is the square root of the sum of the squares of the individual uncertainties

$$\varepsilon_T = \sqrt{\varepsilon_a^2 + \varepsilon_b^2 + \varepsilon_c^2}$$

(Note - this assumes a “normal” distribution (bell curve))

IC Resistor Uncertainty

$$R = \left(\frac{1}{qN_d\mu_n t} \right) \left(\frac{L}{W} \right)$$

Note that N_d , μ_n , t , L , and W are all subject to random variations

The average resistance is found by substituting the averages:

$$\bar{R} = \left(\frac{1}{q\bar{N}_d\bar{\mu}_n\bar{t}} \right) \left(\frac{\bar{L}}{\bar{W}} \right)$$

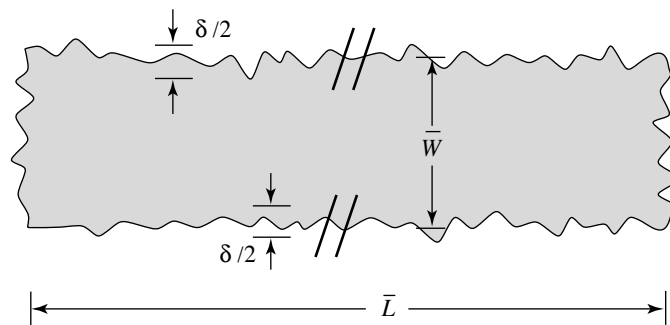
The normalized uncertainty in resistance is found from the “sum of squares” of the normalized uncertainties in N_d , μ_n , t , L , and W

$$\varepsilon_R = \sqrt{\varepsilon_{N_d}^2 + \varepsilon_{\mu_n}^2 + \varepsilon_t^2 + \varepsilon_L^2 + \varepsilon_W^2}$$

This estimate is reasonable for relatively small uncertainties $\varepsilon < 0.1$ and independent variables

Linewidth Uncertainties

- Due to lithographic and etching variation, the edges of a rectangle are “ragged” -
- greatly exaggerated in the figure



- The width is

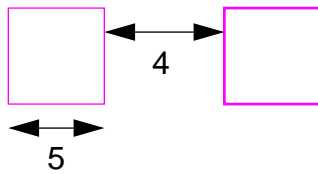
$$W = \bar{W} \pm \frac{\delta}{2} \pm \frac{\delta}{2} = \bar{W} \pm \delta \quad \rightarrow \quad W = \bar{W} \left(1 \pm \frac{\delta}{\bar{W}} \right) = \bar{W} (1 \pm \epsilon_W)$$

- Conclusion 1: *wider* resistors have smaller normalized uncertainty (since δ is independent of width)
- Conclusion 2: the length $L \gg W$ and so its normalized uncertainty is negligible compared to that of W

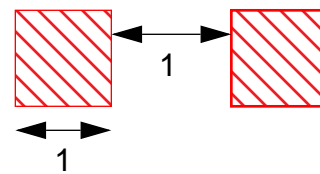
Geometric Design Rules

Uncertainties in the linewidth and the overlay precision of successive masks determines (in part) the rules for laying out masks

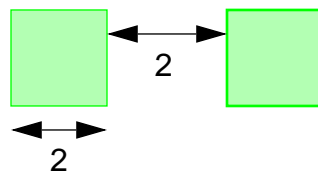
n-well



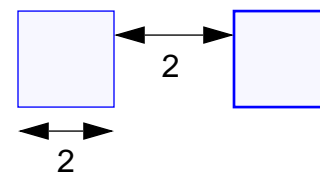
polysilicon



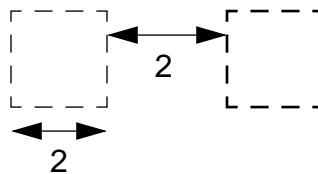
active



metal



select



contact

