Electrostatics: The Key to Understanding Electronic Devices

Physics approach: vector calculus, highly symmetrical problems

Gauss's Law: $\nabla \bullet (\varepsilon \vec{E}) = \rho$

Def. of Potential: $\vec{E} = -\nabla \phi$

Poisson's Eqn.: $\nabla \bullet (\varepsilon(-\nabla \phi)) = \varepsilon \nabla^2 \phi = \rho$

Device physics

Real problems (not symmetrical, complicated boundary conditions)

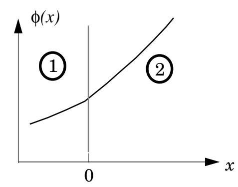
Gauss's Law: $\frac{d(\varepsilon E)}{dx} = \rho$

Definition of Potential: $E = -\frac{d\phi}{dx}$

Poisson's Equation: $\frac{d}{dx}\left(\varepsilon\left(-\frac{d\phi}{dx}\right)\right) = -\varepsilon \frac{d^2\phi}{dx^2} = \rho$

Boundary Conditions

1. Potential: $\phi(x{=}0^-) = \phi(x{=}0^+)$



2. Electric Field: $\varepsilon_1 E(x{=}0^-) + Q = \varepsilon_2 E(x{=}0^+)$

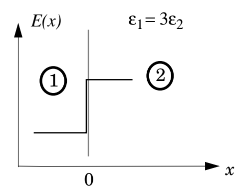
where Q is a surface charge (units, C/cm^2) located at the interface

for the case where $Q = 0$:

common materials:

silicon, $\varepsilon_s = 11.7\ \varepsilon_o$

silicon dioxide (SiO_2), $\varepsilon_{ox} = 3.9\ \varepsilon_o$



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Lecture 6

Intuition for Electrostatics

"Rules of Thumb" for sketching the solution BEFORE doing the math:

- Electric field points from positive to negative charge
- Electric field points "downhill" on a plot of potential
- Electric field is confined to a narrow charged region, in which the positive charge is balanced by an equal and opposite negative charge
- Use boundary conditions on potential or electric field to "patch" together solutions from regions having different material properties
- Gauss's law in integral form relates the electric field at the edges of a region to the charge inside. Often, the field on one side is known to be zero (e.g., because it's on the outside of the charged region), which allows the electric field at an interface to be solved for directly

Charge density functions: only two cases needed for basic device physics

$\rho = 0$ —> E constant --> ϕ linear

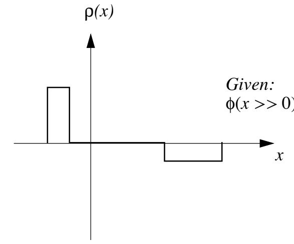
$\rho = \rho_o = $ constant --> E linear --> ϕ quadratic

- surface or sheet charge Q is sometimes present (e.g., on the surface of good conductors); the effect on electric field can be incorporated through the boundary condition

Example I: Applied Electrostatics

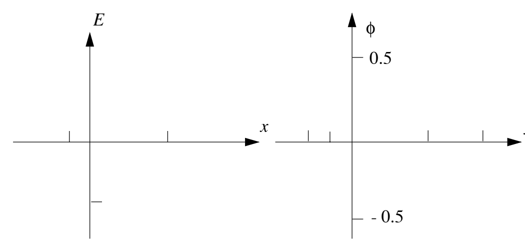
Given:
charge distribution
$\rho(x)$

Given:
$\phi(x << 0) = 0.5$ V



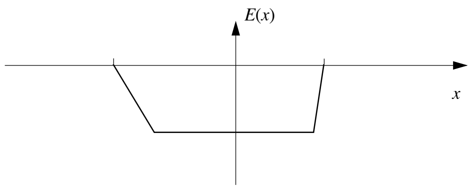
Given:
$\phi(x >> 0) = -0.4$ V

Sketch the electric field and the charge.

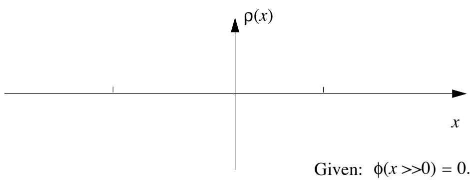


Example II: Applied Electrostatics

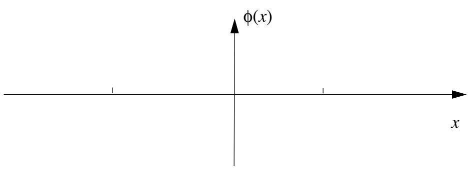
* Given the electric field,



Sketch the charge density and the potential



Given: $\phi(x >> 0) = 0.3$ V



Application of Gauss's Law

- At a point x, the electric field can be found as the charge enclosed, divided by the permittivity of the material ...

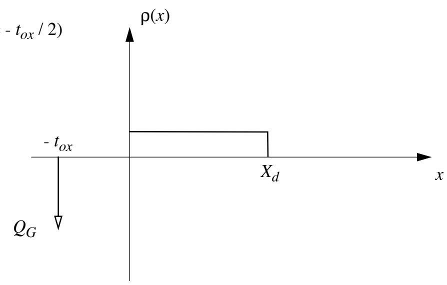
  caveats (warnings):

  (i) the field must be zero at the other side of the charged region

  (ii) the sign of the field can be found by keeping track of the $+x$ direction and the one-dimensional equivalent of the "outward normal;" however, the best approach is to know the sign of the field from the distribution of charge in the problem

- Example: metal-oxide-silicon structure

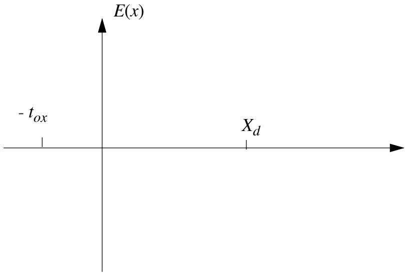
  Find $E(x = -t_{ox} / 2)$



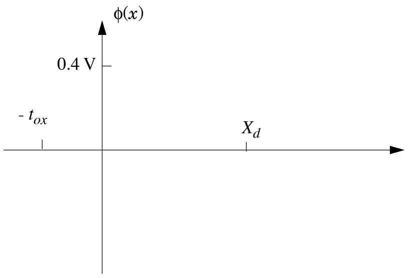
- Find $E(x = 0^+)$ ... just inside the silicon

Boundary Condition on E (cont.)

- Sketch $E(x)$ from $x = -t_{ox}$ to $x = X_d$



- Sketch $\phi(x)$ through the structure, given that $\phi(X_d) = 400$ mV



Potential and Carrier Concentration in Silicon

- What is a convenient reference for the electrostatic potential in silicon?

Thermal equilibrium: no external stimulus --> must have:

$$J_p = 0 \text{ and } J_n = 0.$$

$$\therefore 0 = q n_o \mu_n E + q D_n \frac{dn_o}{dx} = q n_o \mu_n \left(-\frac{d\phi}{dx}\right) + q D_n \frac{dn_o}{dx}$$

$$d\phi = \frac{D_n}{\mu_n}\left(\frac{dn_o}{n_o}\right) = \frac{kT}{q}\left(\frac{dn_o}{n_o}\right) = V_{th}\left(\frac{dn_o}{n_o}\right)$$

where we have used Einstein's relation.

$$\frac{D_n}{\mu_n} = \frac{kT}{q} = V_{th} = 25 \text{ mV at "cool" room temperature to}$$

26 mV at "warm" room temperature

V_{th} is called the *thermal voltage*.

The Intrinsic Potential Reference

- By integrating the equation relating potential to the electron concentration from a position x_a to position x, we find that:

$$\phi(x) - \phi(x_a) = V_{th} \ln\left(\frac{n_o(x)}{n_o(x_a)}\right)$$

We can choose any reference; one convenient choice is to set:

$\phi(x_a) = 0$ when $n_o(x_a) = n_i = 10^{10}$ cm^{-3} at room temperature.

- Using this reference, the potential in thermal equilibrium can be found, given the electron concentration:

$$\phi = V_{th} \ln\left(\frac{n_o}{n_i}\right) = (26\text{mV})\ln(10)\log\left(\frac{n_o}{10^{10}}\right) = (60\text{mV})\log\frac{n_o}{10^{10}}$$

Donor concentrations from 10^{13} to 10^{19} cm^{-3} therefore correspond to potentials of (60 mV) x 3 = 180 mV to (60 mV) x 9 = 540 mV (at room temperature)

The 60 mV Rule

The hole concentration can also be related to the potential, by repeating the derivation starting with $J_p = 0$ or by substituting

$$p_o = n_i^2 / n_o$$

into the 60 mV rule for electrons. The result is:

$$\phi = V_{th} \ln\left(\frac{n_i}{p_o}\right) = (-26\text{mV})\ln(10)\log\left(\frac{p_o}{10^{10}}\right) = (-60\text{mV})\log\left(\frac{p_o}{10^{10}}\right)$$