CHAPTER 3AW

Uncertainty, Risk, and Expected Utility

3AW.1 INTRODUCTION

In the previous chapter, we analyzed rational consumer choice under the assumption that individuals possess perfect information. As a result, we treated the results of consumer choice analysis as certain outcomes. Thus, given a specific utility function and particular amounts of goods consumed by an individual, we can determine with certainty the level, or index, of utility received by the consumer. However, in the real world individuals typically do not possess perfect information when they make their consumption choices. For example, when you decide to buy a new pair of jeans, do you first research all of the types of jeans available in your size? You probably don't. This is largely because the acquisition of information is costly in terms of money and time. Furthermore, in some instances perfect information may not exist when a decision is made. Therefore, we often model rational consumer choice without the stringent assumption of perfect information in an effort to make the analysis more applicable to the real world. In this chapter we will model consumer choice in the absence of perfect information using an expected, or von Neumann–Morgenstern, utility function. In addition, we will apply the concept of expected utility to a variety of real-world consumption choices.

3AW.2 PRELIMINARY DEFINITIONS

We can categorize the environment within which consumers make decisions into one of three states: certainty, uncertainty, or risk. **Certainty** is a condition that *exists when an individual has access to perfect information regarding the outcome of his consumption choice*. In contrast, **uncertainty** is *a situation in which a decision maker does not possesses perfect information nor any probabilities associated with the occurrence of a specific outcome*. The third state, risk, falls between the extreme situations of certainty and uncertainty. **Risk** is a situation that *exists when perfect information is unavailable to a decision maker; however, the probabilities associated with all outcomes are known*. It is important for you to possess at least an elementary understanding of probability theory in order to comprehend the concept of risk and its relationship to expected values and expected utility. Therefore, we will briefly review some key statistical concepts.

Recall from statistics that the probability associated with an event is simply a measure of the likelihood of the occurrence of a particular outcome. In some instances, we objectively know these probabilities since they are based on long-term historical experience. For example, we objectively know that the probability of rolling a three on a single die is 1/6, since the probability of any value coming to the surface is equal to one out of the six unique values represented on the die. However, sometimes we cannot determine the probability of an outcome from past experience. In such a case, we must deduce a subjective probability value. Subjective probabilities are simply an individual's best estimate as to the probability of an event's occurrence, based on the limited information she has available. For example, when a consumer assigns a probability to liking a new product based only on seeing it in an advertisement and on her preferences for similar goods, she is indicating a subjective probability.

Once the probabilities associated with each of the possible outcomes are determined, we can estimate the expected value of any one of these outcomes. Specifically, given some variable, X, that has the ability to take on a number of values, such as X_1, $X_2, X_3, \ldots, X_n$, the **expected value** *of the variable X (also known as the mean value of X) equals the summation of the products of each value of X multiplied by its probability of occurrence*. We can mathematically express the expected (or mean) value of the variable X, denoted $E(X)$, as

$$E(X) = Pr_1X_1 + Pr_2X_2 + Pr_3X_3 + \ldots + Pr_nX_n = \sum_{i=1}^{n} Pr_iX_i,$$

where $Pr_1, Pr_2, Pr_3, \ldots, Pr_n$, represent the probabilities associated with the variable X assuming the values of $X_1, X_2, X_3, \ldots, X_n$, respectively. For example, we can determine the expected value associated with a coin toss game using this formula. Assume the payoff value associated with a heads outcome, denoted X_H, is \$10, while a tails outcome, denoted X_T, pays \$5. Clearly, when you toss a coin, the probability of a heads appearing is the same as that of a tails. Thus, $Pr_H = Pr_T = 0.50$, where Pr_H represents the probability of a heads outcome and Pr_T denotes the probability of a tails outcome. We can compute the expected value of this game as

$$E(X) = Pr_HX_H + Pr_TX_T = (0.50)(\$10) + (0.50)(\$5) = \$7.50.$$

This result means that if the individual plays this game a large number of times, the payoff he is expected to receive is \$7.50. By definition, a **fair game** *is one for which the cost of playing the game is equal to the expected value of the game*. Thus, we would consider this game a fair one if the cost of playing it were \$7.50.

We must emphasize that regardless of whether an individual consumes goods under a state of certainty or one of risk, his goal is to maximize his utility. Therefore, when we model an individual's consumption choices under a state of risk, we must incorporate information regarding expected outcomes and their associated probabilities into the consumer's utility function. Such an **expected utility function**, developed by John von Neumann and Oskar Morgenstern, *measures the expected utility of a set of* n *possible outcomes as the sum of the products of the utility received from each outcome multiplied by its respective probability of occurrence*. We can mathematically express the expected utility function, also known as a von Neumann–Morgenstern utility function, as

$$E[U(Pr_i, X_i)] = Pr_1U(X_1) + Pr_2U(X_2) + Pr_3U(X_3) + \ldots + Pr_nU(X_n) = \sum_{i=1}^{n} Pr_iU(X_i).$$

By applying this formulation for expected utility to different utility functions, we can demonstrate alternative consumer attitudes toward risk.

3AW.3 CONSUMER ATTITUDES TOWARD RISK

Economists typically categorize a consumer's attitude toward risk as either risk averse, risk preferring, or risk neutral. An individual is **risk averse** if the *expected utility she receives from the outcome associated with a risky choice is less than the utility she receives from a certain outcome, which is equal to the expected or mean outcome associated with the risky choice*. We can state this relationship mathematically as

$$E[U(X)] < U[E(X)],$$

where X represents the possible outcomes. In general, the utility function for the risk-averse consumer is a strictly concave function such as the one we have illustrated in panel (A) of Figure 3AW.1. We can observe that the marginal utility associated with higher outcome values, measured as the slope of the utility function, diminishes for this function. A mathematical example of such a utility function in this case is

$$U(X) = X^{1/2},$$

where the consumer's utility, $U(X)$, is a function of only one outcome, X. The associated marginal utility function in this case is

$$MU_X = \frac{dU(X)}{dX} = \frac{1}{2}X^{-1/2},$$

which exhibits diminishing marginal utility for additional units of outcome X. This means that the risk-averse consumer is not willing to incur additional risk for the possibility of a higher valued outcome.

We can demonstrate the behavior of a risk-averse individual using the utility function $U = X^{1/2}$, within the context of the coin toss game we described earlier. As a result, outcome X is again defined as the payoff associated with the game. By definition, if a consumer is risk averse, then the expected utility he receives from playing the game is less than the utility of the expected value of the game, or

$$E[U(X)] = Pr_HU(X_H) + Pr_TU(X_T) < U[E(X)],$$

where

$$U[E(X)] = U(Pr_HX_H + Pr_TX_T).$$

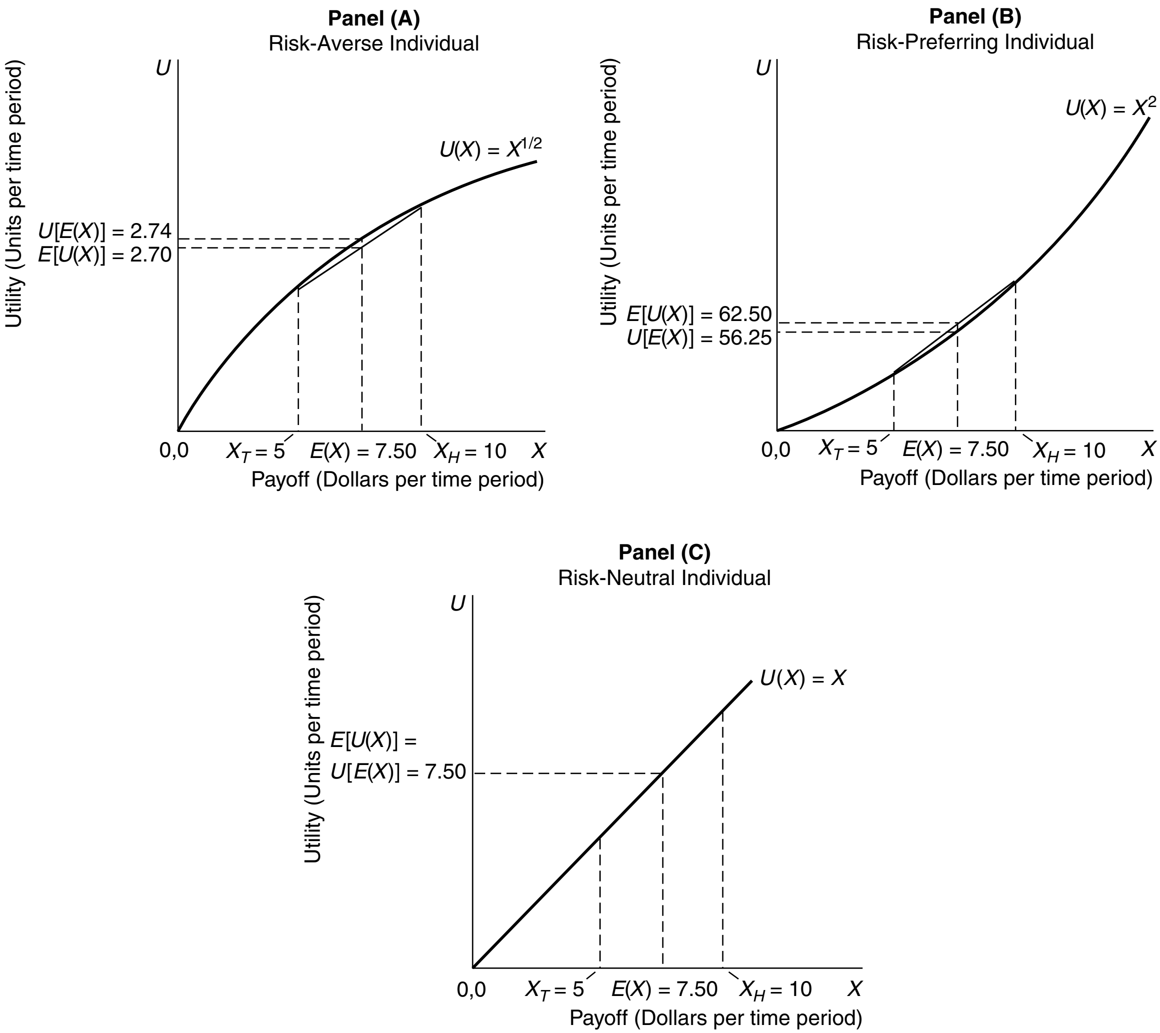


FIGURE 3AW.1 Utility from Coin Toss Game for Different Individuals

Recall that if a game is fair, then the cost of participating in the game must equal its expected value, which for this coin toss game we computed earlier to be \$7.50. Since we assume that the consumer's utility function is

$$U(X) = X^{1/2},$$

then his expected utility from this game, where $X_H = \$10$, $X_T = \$5$, $Pr_H = 0.50$, and $Pr_T = 0.50$, is computed as

$$\begin{aligned} E[U(X)] &= Pr_H U(X_H) + Pr_T U(X_T) = (0.50)X_H{}^{1/2} + (0.50)X_T{}^{1/2} \\ &= (0.50)(10)^{1/2} + (0.50)(5)^{1/2} = 1.58 + 1.12 = 2.70. \end{aligned}$$

The utility he receives from holding on to the \$7.50 he must pay to play the game, which is equal to the expected value of the payoff of the game, is computed as

$$U[E(X)] = U(\$7.50)^{1/2} = 2.74.$$

Since this value is greater than the 2.70 units of utility he would receive from playing the game, this risk-averse consumer logically chooses not to participate in the game but instead would hold on to the \$7.50 he possesses with certainty.

A second category of consumer attitudes toward risk is that of risk preferring. In general, a consumer is said to be **risk preferring** if *the expected utility he receives from the outcome associated with a risky choice is greater than the utility he receives from an outcome with certainty, which is equal to the expected, or mean, outcome associated with the risky choice*. We can state this relationship mathematically as

$$E[U(X)] > U[E(X)].$$

The utility function for a risk-preferring individual is strictly convex, such as the one we have illustrated in panel (B) of Figure 3AW.1. This type of utility function exhibits increasing marginal utility as the outcome value increases. An example of this type of utility function is

$$U(X) = X^2,$$

where once again X represents the value of the outcome. Note that the marginal utility function for outcome X in this case is

$$MU_X = 2X,$$

demonstrating that MU_X increases with increases in X. This means that the risk-preferring consumer is quite willing to take on additional risk for the possibility of a higher valued outcome.

Returning to the coin toss game, a consumer is categorized as risk preferring if the expected utility he receives from playing the game is greater than the utility of the expected value of the game, or

$$E[U(X)] = Pr_H U(X_H) + Pr_T U(X_T) > U[E(X)],$$

where

$$U[E(X)] = U(Pr_H X_H + Pr_T X_T).$$

We can demonstrate this mathematically using the utility function

$$U(X) = X^2,$$

along with the probabilities, payoffs, and expected value of the game we computed earlier. The value of the expected utility function in this case is

$$E[U(X)] = Pr_H U(X_H) + Pr_T U(X_T) = 0.50(10)^2 + 0.50(5)^2 = 62.50,$$

while the utility the consumer receives from the money that he spends to play the game is computed as

$$U[E(X)] = U(X^2) = (7.50)^2 = 56.25.$$

As a result, a risk-preferring individual chooses to participate in the game since he receives more utility from so doing than from simply holding on to the $7.50 it would cost him to play the game.

As a final possibility, a consumer can be characterized as **risk neutral** *if the expected utility he receives from the outcome associated with a risky choice is precisely equal to the utility he receives from an outcome with certainty, which is equal to the expected, or mean, outcome associated with the risky choice*. Mathematically, we express this relationship as

$$E[U(X)] = U[E(X)].$$

The utility function for a risk-neutral individual is, in general, a linear function, such as the one we have illustrated in panel (C) of Figure 3AW.1. This type of utility function exhibits a constant marginal utility with increases in outcome values. An example of such a linear utility function is

$$U(X) = X,$$

for which the MU_X function is

$$MU_X = 1,$$

demonstrating that MU_X is a constant value. This result indicates that the risk-neutral consumer is indifferent with regard to taking on additional risk for the possibility of a higher valued outcome since his marginal utility is unaffected by changes in the amount of risk he assumes.

Returning to the coin toss game, we can categorize a consumer as risk neutral if the expected utility he receives from playing the game is equal to the utility of the expected value of the game, or

$$E[U(X)] = Pr_H U(X_H) + Pr_T U(X_T) = U[E(X)],$$

where

$$U[E(X)] = U(Pr_H X_H + Pr_T X_T).$$

Using the utility function $U(X) = X$, along with the probabilities, payoffs, and expected value of the game that we previously determined, we can compute the value of the expected utility from playing the coin toss game as

$$E[U(X)] = Pr_H U(X_H) + Pr_T U(X_T) = 0.50(10) + 0.50(5) = 7.50.$$

Alternatively, the amount of utility the consumer receives with certainty from the money that he must spend in order to play a fair game, that is, the expected value of the game, $E(X) = 7.50$, is computed as

$$U[E(X)] = U(7.50) = 7.50.$$

In this case, a risk-neutral individual is indifferent with regard to participating in the game since he receives equal utility from either doing so or from holding on to the $7.50 it would cost him to play.

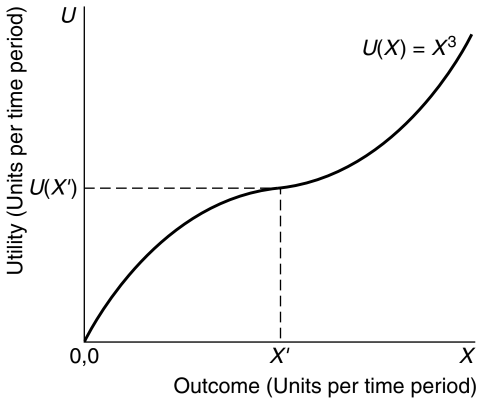


FIGURE 3AW.2 Utility Function for an Individual Who Is Risk Averse with Lower Outcomes and Risk Preferring with Higher Outcomes

In a seminal article in the area of consumer choice under risk, Milton Friedman and Leonard Savage[1] explore the possibility of an individual being both risk averse and risk preferring, depending on the value of the payoff. They suggest that over given ranges of outcomes, an individual's risk preferences may radically shift. In particular, for relatively lower outcomes, the individual may be risk averse, while for higher outcomes he may become risk preferring. In such a case, the individual's utility function is cubic—specifically, concave for relatively lower outcomes, such as those from low-valued lottery jackpots, and convex for relatively higher outcomes from large lottery jackpots. We have illustrated this type of utility function in Figure 3AW.2, for which the individual is risk averse for outcomes of less than X' dollars and risk preferring for outcomes in excess of X' dollars, where the point X' pertains to the inflection point on the curve. From an economic perspective, this is the point on the individual's utility function where the utility associated with an outcome stops increasing at a decreasing rate and begins increasing at an increasing rate.

3AW.4 APPLICATIONS OF CONSUMER CHOICE UNDER RISK

We can apply the notion of consumer choice under risk to more serious considerations than coin toss games. For example, assume an individual has the option of choosing between buying one of two automobiles, specifically, a new Ford Probe or a used Corvette. Based on consumer and automobile industry reports, each car, on average, is expected to provide equal performance, measured in terms of shifting, steering, acceleration, and braking. Hence, the expected, or mean, value of the services rendered by the Ford Probe, denoted $E(X^F)$, is equal to the expected value of the services provided by the used Corvette, $E(X^C)$. However, the two vehicles are expected to differ significantly in terms of the variability of their performance. According to auto industry reports, along with the fact that it is a new vehicle, the Probe is expected to provide

[1]Milton Friedman, and Leonard Savage, "The Utility Analysis of Choices Involving Risk," *Journal of Political Economy*, 56 (1948): 279–304.

extraordinarily consistent, albeit unexciting, operating performance. Thus, under a best case scenario, the service rendered by the new Ford Probe, denoted X_B^F, is equal to that which it provides under a worst case scenario, denoted X_W^F, where both are equal to 200 units as measured by some type of service index. In contrast, the used Corvette, when it is operating at its best, is expected to provide outstanding performance. However, based on the historical service records of other Corvettes of the same vintage, as well as its own past performance, the used Corvette under consideration is known to be far less reliable than the new Ford Probe. Given a best case scenario, the used Corvette delivers outstanding performance, denoted X_B^C, equal to 400 units. However, because at times the Corvette simply doesn't run, its worst possible performance, X_W^C, is zero. In Table 3AW.1 we have summarized the service rendered by each vehicle under the two extreme states of performance, along with the probabilities associated with all possible outcomes for the new Probe and the used Corvette.

Note that we use the superscripts F and C throughout this example to refer to the Ford Probe and the Corvette, respectively, and the subscripts, B and W, respectively, to indicate the best and worst performance scenarios for the automobiles.

Using the information in Table 3AW.1 regarding the service rendered by each automobile under the best and worst case scenarios and the probabilities of their occurrences, we can compute the expected, or mean, value of performance for the new Ford Probe and the used Corvette, respectively, as follows,

$$E(X^F) = Pr_B^F(X_B^F) + Pr_W^F(X_W^F),$$
$$E(X^F) = (0.50)(200) + (0.50)(200) = 200$$

and

$$E(X^C) = Pr_B^C(X_B^C) + Pr_W^C(X_W^C)$$
$$E(X^C) = (0.50)(400) + (0.50)(0) = 200,$$

thus proving, as we stated earlier, that the expected, or mean, level of service rendered by each car is the same.

Since we assume the individual choosing between the new Probe and the used Corvette prefers risk, his utility function must be strictly convex. Let's assume his utility function is

$$U(X) = 10X^2,$$

TABLE 3AW.1 Services Rendered by Two Automobiles

Automobile	***Scenario 1: Best Performance***		***Scenario 2: Worst Performance***		***Expected Value of Services Rendered***
	Probability	***Services Rendered***	***Probability***	***Services Rendered***	
New Ford Probe	$Pr_B^F = 0.50$	$X_B^F = 200$	$Pr_W^F = 0.50$	$X_W^F = 200$	$E(X^F) = 200$
Used Corvette	$Pr_B^C = 0.50$	$X_B^C = 400$	$Pr_W^C = 0.50$	$X_W^C = 0$	$E(X^C) = 200$

where X represents the service rendered by an automobile. Using the data presented in Table 3AW.1 along with this utility function, we can determine the expected utility each automobile yields. In the case of the Ford Probe, the expected utility is computed as

$$E[U(X^F)] = Pr_B^F U(X_B^F) + Pr_W^F U(X_W^F) = 0.50(10(200)^2) + 0.50(10(200)^2) = 400{,}000,$$

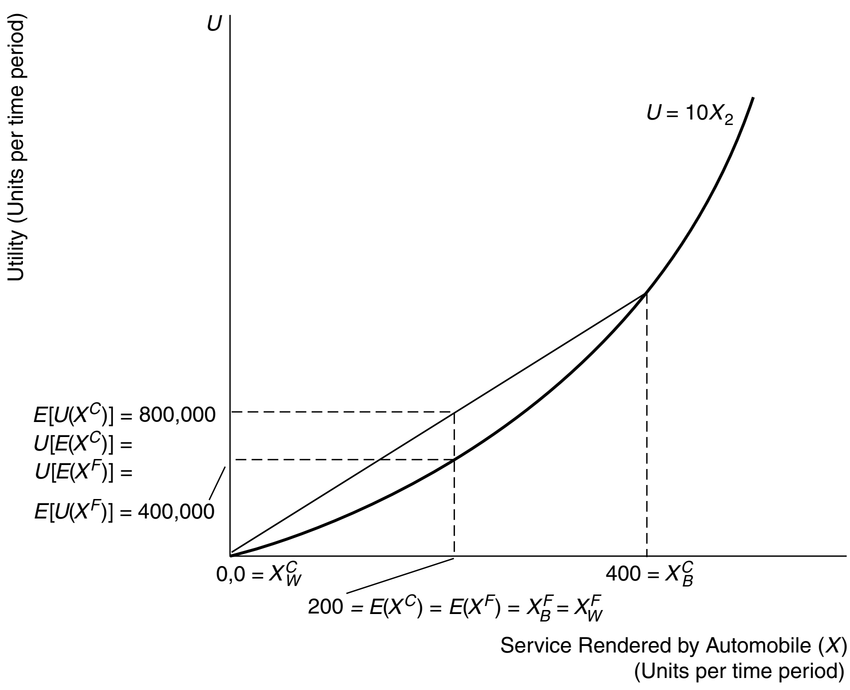
while the expected utility from the used Corvette is determined as

$$E[U(X^C)] = Pr_B^C U(X_B^C) + Pr_W^C U(X_W^C) = 0.50(10(400)^2) + 0.50(10(0)^2) = 800{,}000.$$

In this example, it is clear that an individual seeking to maximize his expected utility will choose to purchase the used Corvette, since its expected utility is twice that of the new Ford Probe. This choice is consistent with the fact that the individual is risk-preferring. He is obviously willing to take the risk of purchasing a car that, at times, does not run in order to enjoy the possibility of the superior driving performance it yields when it operates at its peak form. We have demonstrated this result graphically in Figure 3AW.3 where it is clear that the expected utility from the used Corvette, 800,000, is double the expected utility from the new Probe, 400,000.

An alternative method for assessing consumer choice under risk uses the notion of **variance**, defined as *a measure of the dispersion of a random variable about its mean, or expected value*. We can think of variance as measuring how widely scattered observed values of a random variable are relative to its mean value. Alternatively

FIGURE 3AW.3 Utility from Automobiles Assuming Risk-Preferring Individual



stated, the variance is simply the expected value of the squared differences between the observed values of a random variable and its mean. Mathematically, we can express the variance of the random variable X as

$$\begin{aligned}\text{Var}(X) &= Pr_1[X_1 - E(X)]^2 + Pr_2[X_2 - E(X)]^2 + \ldots + Pr_n[X_n - E(X)]^2 \\ &= \sum_{i=1}^{n} Pr_i[X_i - E(X)]^2,\end{aligned}$$

where $E(X)$ denotes the expected, or mean, value of X and Pr_i represents the probability associated with each of the X_i possible outcomes. The larger the variance associated with a particular choice, the greater the risk associated with it, since the possible values of the outcome display greater dispersion about the mean value.

In our previous consumer car choice example, we demonstrated that a risk-preferring individual chooses to purchase a used Corvette because it yields a higher level of expected utility than the new Ford Probe. We can obtain the same outcome by comparing the variance of each of these vehicles. In the case of the new Ford Probe, the variance in the service it provides is computed as

$$\begin{aligned}\text{Var}(X^F) &= Pr_B^F[X_B^F - E(X^F)]^2 + Pr_W^F[X_W^F - E(X^F)]^2 \\ &= 0.50[200 - 200]^2 + 0.50[200 - 200]^2 \\ &= 0.50(0)^2 + 0.50(0)^2 = 0,\end{aligned}$$

and the variance in the service rendered by the used Corvette is

$$\begin{aligned}\text{Var}(X^C) &= Pr_B^C[X_B^C - E(X^C)]^2 + Pr_W^C[X_W^C - E(X^C)]^2 \\ &= 0.50[400 - 200]^2 + 0.50[0 - 200]^2 \\ &= 0.50(200)^2 + 0.50(-200)^2 = 40{,}000.\end{aligned}$$

The variance associated with the services provided by the used Corvette is substantially greater than that associated with the new Probe.

Returning to Figure 3AW.3, we can see that there is no variation in the performance of the new Ford Probe, since it provides 200 units of service when performing under its best and worst possible conditions, thus resulting in zero variance. However, there is a wide range of service offered by the used Corvette, thereby resulting in its much higher variance. Nevertheless, the expected, or average, level of service from the used Corvette is equal to that of the Probe, or

$$E(X^C) = E(X^F) = 200.$$

This result is consistent with the choice made by the risk-preferring individual in this example. He is willing to assume the higher risk associated with the greater variability in service from the used Corvette in order to receive substantially greater expected utility from driving this car. Conversely, if the individual is risk averse, then his obvious choice is the new Ford Probe since its, albeit lower, performance is perfectly consistent, where Var $(X^F) = 0$. Therefore, it represents a riskless option.

Our final example of consumer choice under risk involves an investment portfolio problem in which we assume there are two alternative financial instruments that have

REAL-WORLD APPLICATION 3AW.1

Lottery Game Rankings: Risk and Return Characteristics

Most of the examples we include in this chapter regarding consumer choice under the condition of a risky payoff are presented within the context of a fair game. Would some consumers choose to engage in unfair games, specifically games for which the expected, or mean, payoff is less than the money they possess with certainty that they use to play the games? The answer is frequently yes for many risk-preferring consumers. This occurs with most types of gambling where the "house," or frequently the state, takes a prescribed cut of the potential payoff. In recent years, probably the most popular form of this type of unfair game has been the state-run lottery. A player purchases a ticket for a certain sum of money, frequently one dollar, and in turn receives the right to claim a potential payoff should a particular set of numbers, drawn by chance, match the numbers on the player's ticket. Since the state automatically withdraws a rather large cut of the overall payoff, the expected value of the player's payoff is substantially less than the money, with certainty, he pays to play the game.

The payoffs and probabilities associated with winning vary across different state lottery games. As a result, the variances associated with the expected payoffs also vary across different lottery games. Professors Thomas Garrett of Kansas State University and Russell Sobel of West Virginia University conducted a survey of several state lottery games. Partial results from their study are presented in the accompanying table, where the column EV lists the expected value per dollar spent on a particular game and the column VAR shows the associated variance for the top five and bottom five state lottery games in the United States in 1995.

Garrett and Sobel's results indicate that many consumers are quite willing to play "unfair" games, where some of these games are considerably less fair than others. The authors of this study also found that, *ceteris paribus*, consumers logically prefer games possessing higher expected payoffs but, interestingly, also prefer those games possessing greater variances. This latter observation is consistent with the fact that most lottery players are risk-preferring individuals.

1995 Lottery Game Rankings: Risk and Return Characteristics

Expected Value and Variance

Top 5 Lottos				***Bottom 5 Lottos***			
STATE	GAME	EV	VAR	STATE	GAME	EV	VAR
1 Delaware	Delaware Lotto	0.61	1839.51	1 Idaho	Hot Lotto*	0.24	1520.91
2 Kentucy	Lotto Kentucky	0.60	8763.11	2 Minnesota	Gopher 5	0.34	17,387.74
3 Massachusetts	Mass Millions	0.55	8128.96	3 New York	Lotto	0.40	201,820.90
4 Multistate	Tri-West Lotto	0.55	20,324.38	4 Ohio	Kicker	0.41	4324.74
5 Wisconsin	Megabucks	0.54	8945.63	5 Montana	Montana Cash	0.45	3328.32

*Idaho's Hot Lotto was terminated on December 27, 1996.

Source: Thomas Garrett and Russell S. Sobel, "Gamblers Favor Skewness, Not Risk: Further Evidence from United States' Lottery Games," *Economic Letters*, 63, No. 1 (April 1999): 85–90.

TABLE 3AW.2 Probabilities and Returns from Mutual Funds

Mutual Fund	*Scenario 1: Bull Market*		*Scenario 2: Bear Market*		*Variance*
	Probability	*Return*	*Probability*	*Return*	
A	$Pr_1^A = 0.60$	$X_1^A = \$10{,}000$	$Pr_2^A = 0.40$	$X_2^A = \$1000$	$\text{Var}\,(X^A) = \$19{,}440{,}000$
B	$Pr_1^B = 0.60$	$X_1^B = \$8{,}000$	$Pr_2^B = 0.40$	$X_2^B = \$4000$	$\text{Var}\,(X^B) = \$3{,}840{,}000$

equal expected, or mean, returns. However, the investments differ significantly with respect to the variability of their returns. Let's assume that an individual has the option of choosing between two mutual funds, A and B. In the interest of simplicity and clarity, throughout this example we will use the superscripts A and B to denote the two mutual funds, and the subscripts 1 and 2 to distinguish between the two alternative states of the stock market, specifically a bull market and a bear market, respectively. We assume that the expected, or mean, returns of the two mutual funds, denoted $E(X^A)$ and $E(X^B)$, respectively, are equal. Using the data in Table 3AW.2, we can compute the expected return from mutual fund A as follows:

$$\begin{aligned} E(X^A) &= Pr_1^A(X_1^A) + Pr_2^A(X_2^A) \\ &= (0.60)(\$10{,}000) + (0.40)(\$1000) \\ &= \$6000 + \$400 = \$6400 \end{aligned}$$

and the expected return from mutual fund B is

$$\begin{aligned} E(X^B) &= Pr_1^B(X_1^B) + Pr_2^B(X_2^B) \\ &= (0.60)(\$8000) + (0.40)(\$4000) \\ &= \$4800 + \$1600 = \$6400, \end{aligned}$$

indicating that the two mutual funds generate, on average, equal expected returns.

The variability associated with each mutual fund, under the two alternative scenarios of a bull market and a bear market, are quite different, however, as we have indicated by the values of the returns of each mutual fund reported in Table 3AW.2. We can compute the variance associated with each mutual fund as follows:

$$\begin{aligned} \text{Var}(X^A) &= Pr_1^A[(X_1^A - E(X^A))^2] + Pr_2^A[(X_2^A - E(X^A))^2] \\ &= (0.60)[(\$10{,}000 - \$6400)^2] + (0.40)[(\$1000 - \$6400)^2] \\ &= (0.60)(\$3600)^2 + (0.40)(\$\,-5400)^2 \\ &= \$7{,}776{,}000 + \$11{,}664{,}000 = \$19{,}440{,}000 \end{aligned}$$

and

$$\begin{aligned} \text{Var}(X^B) &= Pr_1^B[(X_1^B - E(X^B))^2] + Pr_2^B[(X_2^B - E(X^B))^2] \\ &= (0.60)[(\$8000 - \$6400)^2] + (0.40)[(\$4000 - \$6400)^2] \\ &= (0.60)(\$1600)^2 + (0.40)(\$\,-2400)^2 \\ &= \$1{,}536{,}000 + \$2{,}304{,}000 = \$3{,}840{,}000. \end{aligned}$$

The variance associated with the return from mutual fund A is much greater than that associated with mutual fund B. As a result, if the investor is risk preferring, then she will logically choose to purchase shares of mutual fund A. However, if the investor is more risk averse, then she will opt for mutual fund B. To prove this conclusion, we will compute and compare the expected utilities for each of these mutual funds. We will first use a utility function that is consistent with a risk-preferring individual. Later we will use a utility function representative of a risk-averse investor. If we assume a strictly convex utility function, such as

$$U(X) = 2X^2,$$

for a risk-preferring individual, the expected utility from each mutual fund is computed as

$$\begin{aligned} E[U(X^A)] &= Pr_1^A U(X_1^A) + Pr_2^A U(X_2^A) \\ &= (0.60)(2(10{,}000)^2) + (0.40)(2(1000)^2) \\ &= 120{,}000{,}000 + 800{,}000 = 120{,}800{,}000 \end{aligned}$$

and

$$\begin{aligned} E[U(X^B)] &= Pr_1^B U(X_1^B) + Pr_2^B U(X_2^B) \\ &= (0.60)(2(8000)^2) + (0.40)(2(4000)^2) \\ &= 76{,}800{,}000 + 12{,}800{,}000 = 89{,}600{,}000. \end{aligned}$$

Since the expected utility from mutual fund A is far greater than that received from mutual fund B, a risk-preferring individual will choose to invest her money in mutual fund A.

If, however, the investor is risk averse and possesses a utility function such as

$$U(X) = 2X^{1/2},$$

then we can once again compute the expected utility associated with each mutual fund as follows:

$$\begin{aligned} E[U(X^A)] &= Pr_1^A U(X_1^A) + Pr_2^A U(X_2^A) \\ &= (0.60)(2(10{,}000)^{1/2}) + (0.40)(2(1000)^{1/2}) \\ &= 120 + 25.30 = 145.30 \end{aligned}$$

and

$$\begin{aligned} E[U(X^B)] &= Pr_1^B U(X_1^B) + Pr_2^B U(X_2^B) \\ &= (0.60)(2(8000)^{1/2}) + (0.40)(2(4000)^{1/2}) \\ &= 107.33 + 50.60 = 157.96. \end{aligned}$$

In this case, the risk-averse investor will distinctly prefer to invest in mutual fund B since it offers her a higher expected utility than the relatively more risky mutual fund A. We have demonstrated this result in Figure 3AW.4, where we can observe in this example that the consumer's choice is between two risky alternatives, rather than between a risky and a risk-free option, as was the case in our previous automobile example.

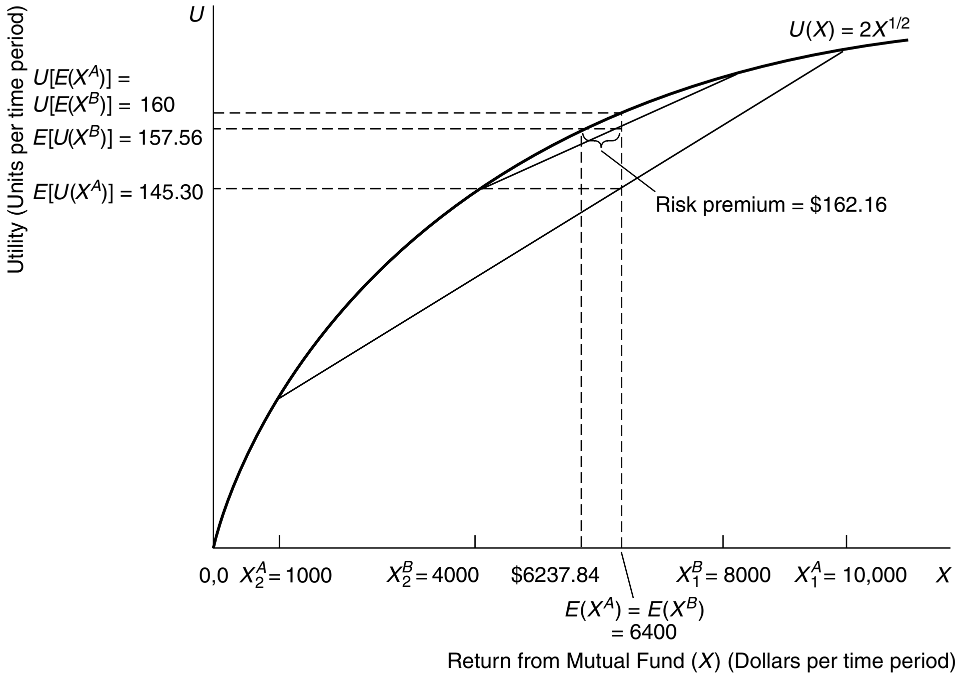


FIGURE 3AW.4 Utility from Mutual Funds Assuming Risk-Averse Individual

An interesting topic related to our discussion of consumer choice under risk is that of a **risk premium**, defined as *the amount of money an individual is willing to forego in order to make him indifferent between a risky investment and one with a certain return*. We can easily apply the concept of a risk premium to the investment choice problem we developed earlier. Recall that if an investor is risk averse, then her utility function is concave. Once again we will assume that the risk-averse individual's utility function is

$$U(X) = 2X^{1/2},$$

where X measures the return an individual receives on an investment. Since we know that the investor is risk averse, it is of interest for us to determine the amount of return the individual needs to receive with certainty from investing in a riskless financial instrument, such as a certificate of deposit, in order to provide her with the same level of utility as the expected return from the moderately risky mutual fund B. To determine this value, recall that we computed the investor's expected utility from investing in mutual fund B to be 157.96. By setting the individual's utility function equal to this level of satisfaction, we can compute the return, X, necessary to yield a utility index level of 157.96 as follows:

$$\begin{aligned} U(X) &= 2X^{1/2} = 157.96 \\ X^{1/2} &= 78.98 \\ (X^{1/2})^2 &= (78.98)^2 \\ X &= \$6237.84. \end{aligned}$$

REAL-WORLD APPLICATION 3AW.2

Rolling the Dice with Mother Nature

The floods generated by Hurricane Floyd in 1999 resulted in millions of dollars of damage to homes and businesses located along the eastern coast of the United States. However, in many instances those individuals whose property was ravaged by the wrath of Hurricane Floyd held no flood insurance, despite the fact that their homes or businesses were located on a flood plain. Such risky behavior is by no means uncommon. Overall, only 25 percent to 50 percent of homeowners located in areas of the United States prone to flooding purchase flood insurance.[2] This is in spite of the fact that flood insurance is required for home buyers located in flood-prone areas who obtain their mortgages from federally regulated lenders. Part of this discrepancy can be attributed to lax enforcement of these insurance requirements on the part of mortgage lenders, while home or business owners who do not hold such mortgages are not required to purchase flood insurance coverage and thus can legally partake of such risky behavior. It is likely, however, that some individuals may view the average annual flood insurance premium of $340 for $120,000 of limited property damage coverage to be too costly and are thus willing to take their chances with Mother Nature. Regardless of the reason, the limited number of flood insurance policies issued in the United States indicates a preference toward risk-taking behavior, especially by those homeowners and businesses whose properties are highly susceptible to flood damage. Such actions indicate that these individuals receive more utility from the $340 that each would have to pay for flood insurance than from the expected utility generated by the insurance claims they would receive if they purchased flood insurance. Conversely, the expected utility received by risk-averse individuals from possible flood insurance claims would be greater than the utility generated from holding on to the $340. Therefore, risk-averse property owners logically seek to minimize their losses by purchasing flood insurance policies.

From the perspective of supply, the market for flood insurance is unique in that the insurance industry in the United States has intentionally shied away from issuing such policies due to the high probability that those individuals purchasing them are likely to file costly claims. As a result, homeowners and small businesses in the United States can obtain flood insurance only through the federally sponsored National Flood Insurance Program. In recent years, flood insurance payouts have been quite costly. It has been estimated that Hurricane Floyd will result in 15,000 to 20,000 claims totaling $300,000 million to $350,000 million, far exceeding the $270,000 million in flood damage that occurred in the U.S. Midwest in 1993. In light of the frequency and magnitude of damaging floods in recent years, it will be interesting to see whether more homeowners choose to bite the bullet and purchase flood insurance or roll the dice with Mother Nature one more time.

[2]Deborah Lohse, "Floyd Teaches a Hard Lesson in Flood Insurance," *The Wall Street Journal*, September 23, 1999, p. B-1.

Thus, the individual in this case is indifferent between receiving a guaranteed return of \$6237.84 or a risk-bearing return of \$6400, the expected return from mutual fund *B*. The difference between these two returns is the risk premium. Specifically, for this example

$$\text{Risk Premium} = \$6400 - \$6237.84 = \$162.16.$$

How can we interpret this value? This risk premium indicates that the individual is willing to forego \$162.16 of the possible \$6400 return she expects, on average, to receive from mutual fund *B* if she is guaranteed a risk-free return of \$6237.84. Figure 3AW.4 illustrates the utility function and risk premium for the individual in this example. The notion of a risk premium is directly applicable to insurance policies. An individual who purchases an insurance policy willingly pays a sum of money, known as an insurance premium, in order to guarantee a certain level of monetary value generally associated with some type of income or asset (see Real-World Application 3AW.2). It should now be abundantly evident to you that we can apply the concept of expected utility to many real-world issues involving consumer choice, such as games of chance, portfolio selection, and insurance and security system purchases.

3AW.5 SUMMARY

In this chapter we applied the concept of risk to traditional consumer theory using expected utility functions, also known as von Neumann–Morgenstern utility functions. Afterward, we explored several real-world applications of consumer choice under risk. The key topics we discussed in this chapter are as follows.

- The expected, or mean, value of a variable, X, that has the ability to take on a number of values, such as $X_1, X_2, \ldots, X_n$, equals the summation of the products of each value of X multiplied by its probability of occurrence.
- A fair game is one for which the cost of playing the game is equal to the expected, or mean, value of the game.
- If we relax the assumption of perfect information on the part of the individual, we can model rational consumer behavior under risk using a von Neumann–Morgenstern, or expected, utility function where a consumer maximizes the expected utility he derives from a set of n possible outcomes.
- An individual is risk averse if the expected utility he receives from an outcome associated with a risky choice is less than the utility he receives from an outcome with certainty, which is equal to the expected, or mean, outcome associated with the risky choice.
- A consumer is risk preferring if the expected utility he receives from an outcome associated with a risky choice is greater than the utility he receives from an outcome with certainty, which is equal to the expected, or mean, outcome associated with the risky choice.
- A consumer can be characterized as risk neutral if the expected utility he receives from the outcome associated with a risky choice is precisely equal to

the utility he receives from an outcome with certainty, which is equal to the expected, or mean, outcome associated with the risky choice.

- A risk premium measures the amount of money an individual is willing to forego from investing in a risky instrument in order to make him indifferent between a risky investment and one with a riskless return.
- We can apply expected utility analysis to various real-world situations, including investment portfolio selection, purchases of insurance, games of chance, and choice among risky goods and services.

KEY TERMS

- certainty, page W81
- expected utility function, page W82
- expected value, page W82
- fair game, page W82
- risk, page W81
- risk-averse, page W83
- risk-neutral, page W86
- risk-preferring, page W85
- risk premium, page W94
- uncertainty, page W81
- variance, page W89

EXERCISES

3AW.1 Assume that an individual has a financial portfolio consisting of several types of assets. By altering the types of assets comprising her portfolio, she is able to choose various rates of return that are directly related to associated levels of risk, meaning that the greater the level of risk, the greater is the rate of return. Also assume that this individual's level of utility is inversely related to the level of risk.

a. Designating the rate of return and the amount of risk associated with this financial portfolio as constituting the two goods in her utility function, how might they be described in terms of their respective marginal utilities?

b. Graph the associated indifference curve for these two goods and explain why it assumes the observed shape.

3AW.2 Herb is selling life insurance and is desperately trying to meet his monthly sales quota. He only has time to make one more sales call. Les, Jennifer, and Johnny are all the same age, in excellent health, with identical family responsibilities. However, each has a different utility function for life insurance, where X represents each consumer's wealth. Specifically, Les's utility function is $U(X) = 10X^{.5}$, Jennifer's utility function is $U(X) = 10X$, and Johnny's utility function is specified as $U(X) = 10X^2$. Who should Herb call and why?

3AW.3 Using the data in Table 3AW.1 in Section 3AW.4, determine which car an individual would purchase if his utility function was $U(X) = 5X^{.5}$.

3AW.4 Using the data in Table 3AW.2 in Section 3AW.4, determine which mutual fund an individual would choose to invest his money in if his utility function were $U(X) = 4X^{1/2}$.

3AW.5 Using the data in Table 3AW.2 in Section 3AW.4 along with your response to exercise 3AW.4, determine the value of the risk premium and interpret its meaning to the investor.

3AW.6 Refer to the expected value data reported for various 1995 lottery games in Real-World Application 3AW.1.

a. If the cost of a lottery ticket for each of the state lotteries was $1.00, could any of these lotteries be considered fair games?

b. What type of individual is likely to purchase these lottery tickets?

3AW.7 Samantha and Matt each own homes located on the flood-prone outer banks of North Carolina. These homes represent each person's sole wealth. Samantha's utility function is $U(X) = 2X^2$ and Matt's utility function is $U(X) = 0.5X^{.5}$, where X represents the level of an individual's wealth. Which individual is more likely to purchase flood insurance for his or her home? Explain your response.