Differential Amplifier with Single-Ended Outputs

- An output voltage referenced to ground is important in some applications.

Simple approach: take the output from one side.

Purely differential input voltage --> \( v_{o2} = -\frac{v_{od}}{2} = -(1/2)a_{dm}v_{id} \)

\[
\frac{v_o}{v_{id}} = -\left(\frac{1}{2}\right)(-g_mR_C) = \frac{g_mR_C}{2}
\]

Sign change (since \( v_{p2} = -\frac{v_{id}}{2} \)) and a loss of 50% of gain.
Differential Amplifier with Current Supplies

- Boost gain by using current supplies adjusted to $I_{BIAS}/2$ instead of $R_C$

\[ a_{dm} = -g_m(r_o||r_{oc}) \] for this differential amplifier

Drawbacks:
- Bias stability is not possible without a feedback circuit
- Taking the output from one side still reduces the gain by 50%
Differential Amplifier with Current Mirror Supply

- By substituting a current mirror (diode voltage source biasing a current source transistor), this amplifier has a stable bias.

The output node should be held at a constant DC potential

\[ V_{OUT} = V^+ - V_{SG3} \]

so that the amplifier is balanced and the output is a small-signal short-circuit

- Note that this amplifier is not symmetrical and that half circuits do not apply.
**Short-Circuit Transconductance** \( g_{md} \)

- Approximate circuit analysis:

\[
\begin{align*}
i_D^1 &= I_D^1 + g_m v_{gs1} = \frac{I_{BIAS}}{2} + g_m v_{gs1} \\
i_D^2 &= I_D^2 + g_m v_{gs2} = \frac{I_{BIAS}}{2} + g_m v_{gs2}
\end{align*}
\]

Current mirror forces the drain current \(-i_D^4 = -i_D^3 = i_D^1\)

Kirchhoff’s current law at the output states that

\[i_O = i_D^2 - (-i_D^4) = i_D^2 - i_D^1\]

\[i_O = \left(\frac{I_{BIAS}}{2} + g_m v_{gs2}\right) - \left(\frac{I_{BIAS}}{2} + g_m v_{gs1}\right) = g_m (v_{gs2} - v_{gs1})\]

Kirchhoff’s voltage law at the input states that \(v_{gs2} - v_{gs1} = v_{i2} - v_{i1} = -v_{id}\)

\[i_o = g_m (-v_{id}) \rightarrow \quad G_{md} = \frac{i_o}{v_{id}} = -g_m\]

- No factor of two in converting differential input into a single-ended current!
Output Resistance $R_{od}$

- The current-mirror circuit is not symmetrical, so the procedure must be applied to the entire amplifier.

- Complicated analysis (see Section 11.5), but a simple result:

$$R_{od} = r_{o2} || r_{o4}$$
Two-Port Differential Model: Current-Mirror Supply

- The output port is referenced to ground, in contrast to the earlier model of the symmetrical amplifier with $v_o = v_{od}$
Input Common-Mode Voltage Range

- The range of DC common-mode inputs over which the differential amplifier can function is an important practical specification (see op amp spec. sheets)

Upper limit to $V_{IC}$

devices 1 and 2 leave their constant-current regions

Lower limit to $V_{IC}$

bias current device 3 leaves its constant-current region
All-Bipolar Differential Amplifier $V_{IC}$ Range

- Maximum common-mode input voltage:

$$V_{O1} = V^+ - \left(\frac{I_{BIAS}}{2}\right)R_C$$

$Q_1$ enters saturation when $V_{BC1} = V_{BE1} - V_{CE(sat)1} = 0.7 \text{ V} - 0.1 \text{ V} = 0.6 \text{ V}$

$$V_{IC(max)} = V_{O1} + 0.6 \text{ V} = V^+ - \left(\frac{I_{BIAS}}{2}\right)R_C + 0.6 \text{ V}$$

- Minimum common-mode input voltage:

$$V_X = V_{IC} - V_{BE1} = V_{IC} - 0.7 \text{ V}$$

$Q_3$ enters saturation when $V_X - V^- = V_{CE(sat)3} = 0.1 \text{ V}$

$$V_{IC(min)} = V_X + V_{BE1} = V^- + V_{CE(sat)3} + V_{BE1} = V^- + 0.8 \text{ V}$$
Large-Signal Response of MOS Differential Amplifiers

- MOS differential amplifier

Kirchhoff’s voltage law around input loop

\[ V_{ID} = V_{GS1} - V_{GS2} \]

For a sufficiently large positive differential input voltage, all of the current \( I_{BIAS} \) will be sunk through \( M_1 \) and \( M_2 \) will be cutoff.
Large-Signal Response of MOS Differential Amplifiers

- Solve for this critical value $V_{ID}^*$ by setting $I_{D1} = I_{BIAS}$ and $I_{D2} = 0$ A

\[
V_{ID}^* = \left( V_{Tn} + \sqrt{\frac{I_{BIAS}}{(W/2L)\mu_n C_{ox}}} \right) - V_{Tn} = \sqrt{\frac{I_{BIAS}}{(W/2L)\mu_n C_{ox}}}
\]

- For $|V_{ID}| \leq V_{ID}^*$ we can solve for the transfer function

\[
I_{D1} = K_n (V_{GS1} - V_{Tn})^2 \quad \text{and} \quad I_{D2} = K_n (V_{GS2} - V_{Tn})^2
\]

where $K_n = (1/2)\mu_n C_{ox}(W/L)$

- Solving for $V_{GS1} - V_{GS2} = V_{ID}$

\[
\sqrt{I_{D1}} - \sqrt{I_{D2}} = \sqrt{K_n (V_{GS1} - V_{GS2})} = \sqrt{K_n V_{ID}}
\]

procedure:

use this equation and $I_{D1} + I_{D2} = I_{BIAS}$

solve for $I_{D1}$ and $I_{D2}$ as functions of $V_{ID}$
Large-Signal Transfer Function for MOS Differential Amplifier

- Transition width is adjustable via $W/L$ and $I_{BIAS}$

\[ V_{OUT} \]

\[ V_{O1} \quad V_{O2} \quad V^+ \quad V^+ - \frac{I_{BIAS} R_D}{2} \quad V^+ - I_{BIAS} R_D \]

\[ V_{ID} \]

\[ - \sqrt{\frac{I_{BIAS}}{W L \mu_n C_{ox}}} \quad \frac{I_{BIAS}}{W L \mu_n C_{ox}} \]
Large-Signal Response of Bipolar Differential Amplifiers

- Find large-signal transfer curves for collector currents $I_{C1}$ and $I_{C2}$ and output voltages $V_{O1}$ and $V_{O2}$ as functions of $V_{ID}$.

\[ V_{ID} = \frac{V_{ID}}{2} \]
\[ V_{BE1} = -V_{BE} \]
\[ V_{BE2} = -V_{BE} \]
\[ V_{I1} = \frac{V_{ID}}{2} \]
\[ V_{I2} = -\frac{V_{ID}}{2} \]
Quantitative Large-Signal Model

- \( V_{ID} = V_{BE1} - V_{BE2} \)

- Ebers-Moll for the forward-active region:
  \[
  I_{C1} = I_s e^{V_{BE1}/V_{th}} = I_s e^{(V_{I1} - V_E)/V_{th}}
  \]
  \[
  I_{C2} = I_s e^{V_{BE2}/V_{th}} = I_s e^{(V_{I2} - V_E)/V_{th}}
  \]

  Dividing the two equations, the emitter voltage \( V_E \) can be eliminated:
  \[
  \frac{I_{C2}}{I_{C1}} = e^{(V_{I1} - V_{I2})/V_{th}} = e^{V_{ID}/V_{th}}
  \]

  Since the two emitter currents must sum to equal the bias current \( I_{BIAS} \), the collector currents are also related by:
  \[
  \left(\frac{1}{\alpha_F}\right)(I_{C1} + I_{C2}) = I_{BIAS}
  \]
Large-Signal Differential Response

- Solving for each current as a function of the differential input voltage $V_{ID} = V_{I1} - V_{I2}$:

  $I_{C1} = \frac{\alpha_F I_{BIAS}}{1 + e^{-V_{ID}/V_{th}}}$

  $I_{C2} = \frac{\alpha_F I_{BIAS}}{1 + e^{V_{ID}/V_{th}}}$

- Output voltages:

  $V_{O1} = V^+ - \frac{\alpha_F I_{BIAS} R_C}{1 + e^{-V_{ID}/V_{th}}}$

  $V_{O2} = V^+ - \frac{\alpha_F I_{BIAS} R_C}{1 + e^{V_{ID}/V_{th}}}$
Transfer Functions for Bipolar Differential Amplifier

- Width of transition region

look at current ratio in base 10 --

\[
\frac{I_{C2}}{I_{C1}} = 10^{\frac{V_{ID}}{60 \text{ mV}}} \quad \rightarrow \quad V_{ID} = (60 \text{ mV}) \log \left( \frac{I_{C2}}{I_{C1}} \right)
\]

factor of 10 difference \(\rightarrow V_{ID} = 60 \text{ mV} \) ... practically, \(\pm 3V_{th}\) will swing the output voltage between the limiting values

\[V_{OUT} = V_{O1} + \left( V_{O2} - V_{O1} \right) \frac{-V_{ID}}{2V_{th}} \]

\[V_{OUT} = V_{O1} + \left( V_{O2} - V_{O1} \right) \frac{-V_{ID}}{2V_{th}} \]

\[\text{for } V_{ID} \leq V_{th} \]

\[\text{for } V_{ID} > V_{th} \]

\[\text{for } V_{ID} > 2V_{th} \]